Three Dimensional Geometry

Question1

Let $\mathrm{L}_1: \frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\mathrm{L}_2: \frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}, \alpha \in \mathbf{R}$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point $A(1,1,-1)$ on L_2, then the value of $26 \alpha(\mathrm{PB})^2$ is _____ .

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Answer: 216

Solution:

Explanation

To find the value of $26\alpha(\mathrm{PB})^2$, we proceed as follows:

Intersection Point B of L_1 and L_2

We are given the equations of the lines:

$\mathrm{L}_1: \frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$

$\mathrm{L}_2: \frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}$

For L_1, $z+1=0$, which implies $z=-1$.

For L_2, since $y=0$, the direction ratios can be matched using the parameter μ:

$$x=3\lambda+1, \quad y=-\lambda+1, \quad z=-1,$$
$$x=2\mu+2, \quad y=0, \quad z=\alpha\mu-4.$$

Setting the coordinates equal for intersection:

$$3\lambda+1=2\mu+2,$$
$$-\lambda+1=0,$$
$$-1=\alpha\mu-4.$$

From $-\lambda + 1 = 0$, we find $\lambda = 1$.

Substituting $\lambda = 1$ into $3\lambda + 1 = 2\mu + 2$ gives:

$3(1) + 1 = 2\mu + 2 \implies \mu = 1$.

Also, substituting $\mu = 1$ into $-1 = \alpha\mu - 4$ gives:

$-1 = \alpha(1) - 4 \implies \alpha = 3$.

So, the intersection point B is:

$B(4, 0, -1)$.

Foot of Perpendicular from A to L_2

The point P on L_2 is given by:

$P(2\delta + 2, 0, 3\delta - 4)$.

For vector $\overrightarrow{AP} = (2\delta + 1, -1, 3\delta - 3)$, since AP is perpendicular to L_2, the dot product should be zero:

$(2\delta + 1) \cdot 2 + (-1) \cdot 0 + (3\delta - 3) \cdot 3 = 0$.

Simplifying gives:

$4\delta + 2 + 9\delta - 9 = 0 \implies 13\delta = 7 \implies \delta = \frac{7}{13}$.

So, the coordinates of point P are:

$P\left(\frac{40}{13}, 0, \frac{-31}{13}\right)$.

Distance PB and Calculation

The vector $\overrightarrow{PB}$ is:

$\overrightarrow{PB} = \left(4 - \frac{40}{13}, 0 - 0, -1 - \left(\frac{-31}{13}\right)\right)$.

Calculating the components:

$\overrightarrow{PB} = \left(\frac{12}{13}, 0, \frac{18}{13}\right)$.

The square of the distance $(PB)^2$ is:

$\left(\frac{12}{13}\right)^2 + (0)^2 + \left(\frac{18}{13}\right)^2 = \frac{144}{169} + \frac{324}{169} = \frac{468}{169}$.

Thus, $26\alpha(PB)^2$ is:

$26 \times 3 \times \frac{468}{169} = 216$.

So, the final value is 216.

Question2

Let P be the image of the point $\mathrm{Q}(7,-2,5)$ in the line $\mathrm{L}: \frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ and $\mathrm{R}(5, \mathrm{p}, \mathrm{q})$ be a point on L. Then the square of the area of $\triangle PQR$ is _____.

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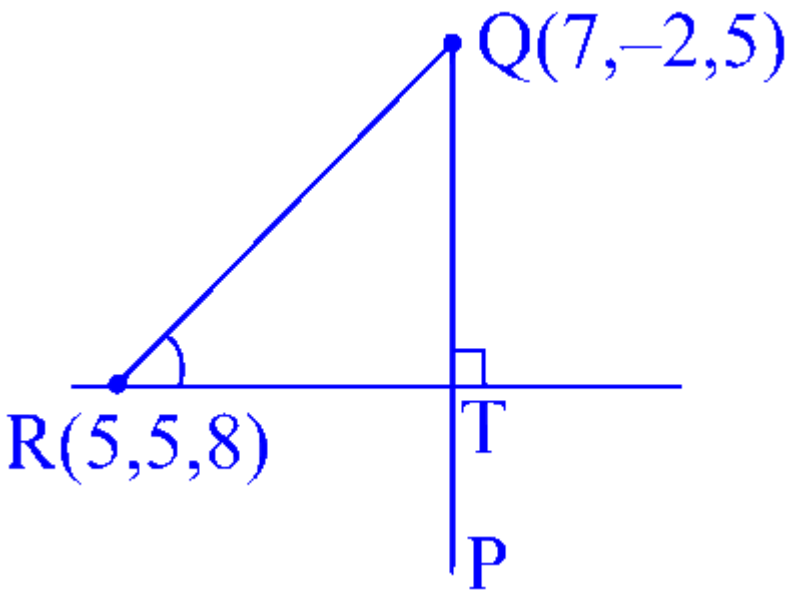
Answer: 957

Solution:



$\text{Let } \mathrm{R}(2\lambda+1, 3\lambda-1, 4\lambda)$

$2\lambda+1=5$

$\lambda=2$

$\mathrm{R}(5,5,8)$

$\text{let } \mathrm{T}(2\lambda+1, 3\lambda-1, 4\lambda)$

$\overrightarrow{\mathrm{QT}}=(2\lambda-6)\hat{\mathrm{i}}+(3\lambda+1)\hat{\mathrm{j}}+(4\lambda-5)\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{b}}=2\hat{\mathrm{i}}+3\hat{\mathrm{j}}+4\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{QT}} \cdot \overrightarrow{\mathrm{b}}=0$

$4\lambda-12+9\lambda+3+16\lambda-20=0$

$\lambda=1$

$\mathrm{T}(3,2,4)$

$\mathrm{QT}=\sqrt{33} \quad \mathrm{RT}=\sqrt{29}$

$(\text{ area of } \Delta \mathrm{PQR})^2=\left(\frac{1}{2}\sqrt{29} \cdot 2\sqrt{33}\right)^2$

$=957$

Question3

Let the area of the triangle formed by the lines $x+2=y-1=z$, $\frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}$ and $\frac{x}{-3}=\frac{y-3}{3}=\frac{z-2}{1}$ be A. Then A^2 is equal to _____.

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Answer: 56

Solution:

$$L_1: x+2=y-1=z=\ell$$
$$L_2: \frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}=m$$
$$L_3: \frac{x}{-3}=\frac{y-3}{5}=\frac{z-2}{1}=n$$

Point of intersection of L_1 and L_2

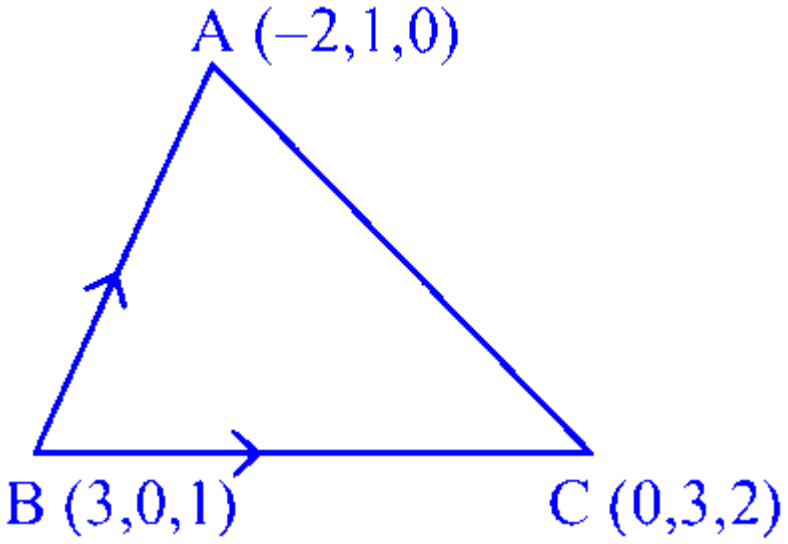
$$\left.\begin{aligned}\ell-2&=5\,\mathrm{m}+3\\ \ell+1&=-\mathrm{m}\\ \ell&=\mathrm{m}+1\end{aligned}\right\}\ell=0,\ \mathrm{m}=-1\quad \mathrm{A}(-2,1,0)$$

Point of intersection of L_2 and L_3

$$\left.\begin{aligned}5m+3&=-3n\\ -m&=3n+3\\ m+1&=n+2\end{aligned}\right\}m=0,n=-1,B(3,0,1)$$

Point of intersection L_3 and L_4

$$\left.\begin{aligned}-3\mathrm{n}&=\ell-2\\ 3\mathrm{n}+3&=\ell+1\\ \mathrm{n}+2&=\ell\end{aligned}\right\}\ell=2,\mathrm{n}=0,\mathrm{C}(0,3,2)$$



$$\text{Ar}(\triangle ABC) = \left|\frac{1}{2}\right| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{matrix} ||$$

$$A = \frac{1}{2}|\hat{i}(4) - \hat{j}(-8) + \hat{k}(-12)|$$

$$A = \frac{1}{2}\sqrt{16 + 64 + 144} = \sqrt{56}$$

$$A^2 = 56$$

Question4

Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

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Options:

A. $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

B. $\left(2, 3, \frac{1}{3}\right)$

C. $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$

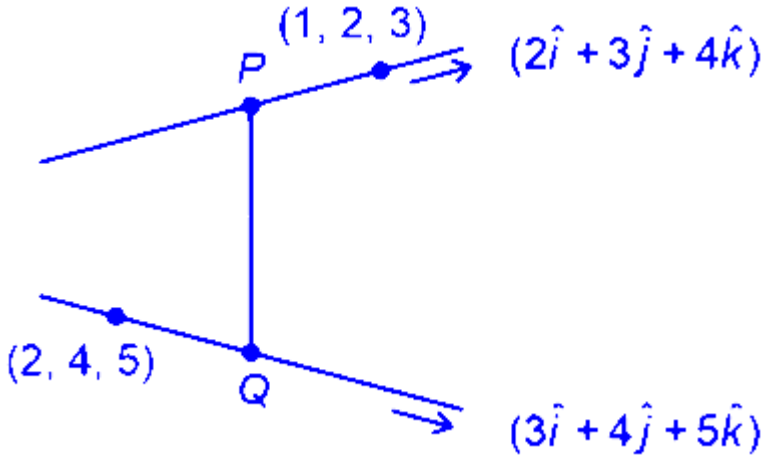
D. $\left(-\frac{5}{3}, -7, 1\right)$

Answer: A

Solution:

$L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

$L_2 : \dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$



$P(2\lambda+1, 3\lambda+2, 4\lambda+3)$
$Q(3\mu+2, 4\mu+4, 5\mu+5)$

Dr's of $PQ < 2\lambda - 3\mu - 1, 3\lambda - 4\mu - 2,$

$4\lambda - 5\mu - 2 >$

$$PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\Rightarrow \quad \frac{2\lambda - 3\mu - 1}{-1} = \frac{3\lambda - 4\mu - 2}{2} = \frac{4\lambda - 5\mu - 2}{-1}$$

$$\Rightarrow \quad \lambda = \frac{1}{3}\mu = \frac{-1}{6}$$

$$\Rightarrow \quad P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$$

Dr's $PQ\langle 1, -2, 1\rangle$

$\therefore$ Line

$\frac{y-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{y-\frac{13}{3}}{1}$

Question5

The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $\mathrm{P}(2, -10, 1)$, is :

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Options:

A. 6

B. $4\sqrt{3}$

C. $3\sqrt{5}$

D. $5\sqrt{2}$

Answer: C

Solution:

To find the perpendicular distance from the point $P(2, -10, 1)$ to the line given by

$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$,

follow these steps:

Parametrize the Line:

From the given symmetric equations, set the common parameter as t:

$x = 1 + 2t$

$y = -2 - t$

$z = -3 + 2t$

This shows that:

A point on the line is $A(1, -2, -3)$ (when $t = 0$).

The direction vector is $\vec{d} = \langle 2, -1, 2 \rangle$.

Determine the Vector from Point A to P:

Calculate

$\vec{AP} = P - A = \langle 2 - 1,\ -10 - (-2),\ 1 - (-3) \rangle = \langle 1,\ -8,\ 4 \rangle$.

Compute the Cross Product:

The formula for the perpendicular distance from a point to a line in 3D is:

$d = \frac{\|\vec{AP} \times \vec{d}\|}{\|\vec{d}\|}$.

First, find the cross product $\vec{AP} \times \vec{d}$:

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -8 & 4 \\ 2 & -1 & 2 \end{vmatrix} = \Big((-8)(2) - (4)(-1),\ (4)(2) - (1)(2),\ (1)(-1) - (-8)(2) \Big).$$

Evaluate each component:

First component: $(-8 \times 2) - (4 \times -1) = -16 + 4 = -12$.

Second component: $(4 \times 2) - (1 \times 2) = 8 - 2 = 6$.

Third component: $(1 \times -1) - (-8 \times 2) = -1 + 16 = 15$.

So,

$\vec{AP} \times \vec{d} = \langle -12, 6, 15 \rangle$.

Calculate the Magnitudes:

For the cross product:

$$\|\vec{AP} \times \vec{d}\| = \sqrt{(-12)^2 + 6^2 + 15^2} = \sqrt{144 + 36 + 225} = \sqrt{405} = 9\sqrt{5}.$$

For the direction vector:

$$\|\vec{d}\| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

Compute the Distance:

Substitute the magnitudes into the distance formula:

$d = \frac{9\sqrt{5}}{3} = 3\sqrt{5}$.

Thus, the perpendicular distance from the point $P(2, -10, 1)$ to the line is $3\sqrt{5}$.

Comparing with the options given, the correct answer is:

Option C: $3\sqrt{5}$.

Question6

Let a line pass through two distinct points $P(-2, -1, 3)$ and Q, and be parallel to the vector $3\hat{i} + 2\hat{j} + 2\hat{k}$. If the distance of the point Q from the point $\mathrm{R}(1, 3, 3)$ is 5 , then the square of the area of $\triangle PQR$ is equal to :

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Options:

A. 148

B. 144

C. 136

D. 140

Answer: C

Solution:

$P = (-2, -1, 3)$

Since the line through P is parallel to the vector

$\vec{d} = (3, 2, 2)$,

any point Q on this line can be expressed as:

$Q = P + t\,\vec{d} = (-2 + 3t,\ -1 + 2t,\ 3 + 2t)$,

where t is a real number. Given that P and Q must be distinct, we require $t \neq 0$.

The point R is given by:

$R = (1, 3, 3)$.

The distance from Q to R is 5, so we have:

$\left[(-2 + 3t - 1)^2 + (-1 + 2t - 3)^2 + (3 + 2t - 3)^2\right] = 5^2$.

Simplify each coordinate difference:

For the x-coordinate:

$-2 + 3t - 1 = 3t - 3 = 3(t - 1)$.

For the y-coordinate:

$-1 + 2t - 3 = 2t - 4 = 2(t - 2)$.

For the z-coordinate:

$3 + 2t - 3 = 2t$.

So the equation becomes:

$[3(t - 1)]^2 + [2(t - 2)]^2 + (2t)^2 = 25$.

Expanding the squares:

$9(t - 1)^2 + 4(t - 2)^2 + 4t^2 = 25$.

Expand each term:

$9(t^2 - 2t + 1) + 4(t^2 - 4t + 4) + 4t^2 = 25,$

which simplifies to:

$9t^2 - 18t + 9 + 4t^2 - 16t + 16 + 4t^2 = 25.$

Combine like terms:

$(9t^2 + 4t^2 + 4t^2) - (18t + 16t) + (9 + 16) = 25,$

$17t^2 - 34t + 25 = 25.$

Subtract 25 from both sides:

$17t^2 - 34t = 0.$

Factor out $17t$:

$17t(t - 2) = 0.$

Since $t \neq 0$, we must have:

$t = 2.$

Substitute $t = 2$ back into the equation for Q:

$Q = (-2 + 3(2),\ -1 + 2(2),\ 3 + 2(2)) = (4, 3, 7).$

Next, to find the square of the area of triangle PQR, first compute the vectors:

$\vec{PQ} = Q - P = (4 - (-2),\ 3 - (-1),\ 7 - 3) = (6, 4, 4),$

$\vec{PR} = R - P = (1 - (-2),\ 3 - (-1),\ 3 - 3) = (3, 4, 0).$

The area of the triangle is given by:

Area $= \frac{1}{2} \|\vec{PQ} \times \vec{PR}\|.$

Calculate the cross product:

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}.$$

Expanding the determinant:

$\hat{i}$-component:

$4 \times 0 - 4 \times 4 = -16,$

$\hat{j}$-component:

$-(6 \times 0 - 4 \times 3) = 12,$

$\hat{k}$-component:

$6 \times 4 - 4 \times 3 = 24 - 12 = 12.$

Thus,

$\vec{PQ} \times \vec{PR} = (-16,\ 12,\ 12)$.

Find the magnitude squared of the cross product:

$\|\vec{PQ} \times \vec{PR}\|^2 = (-16)^2 + 12^2 + 12^2 = 256 + 144 + 144 = 544$.

The square of the area of triangle PQR is:

$(\text{Area})^2 = \left(\frac{1}{2}\right)^2 \|\vec{PQ} \times \vec{PR}\|^2 = \frac{1}{4} \times 544 = 136$.

Thus, the square of the area of $\triangle PQR$ is 136.

Question7

Let P be the foot of the perpendicular from the point $\mathrm{Q}(10, -3, -1)$ on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point $(3, -2, 1)$, is

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Options:

A. $\sqrt{30}$

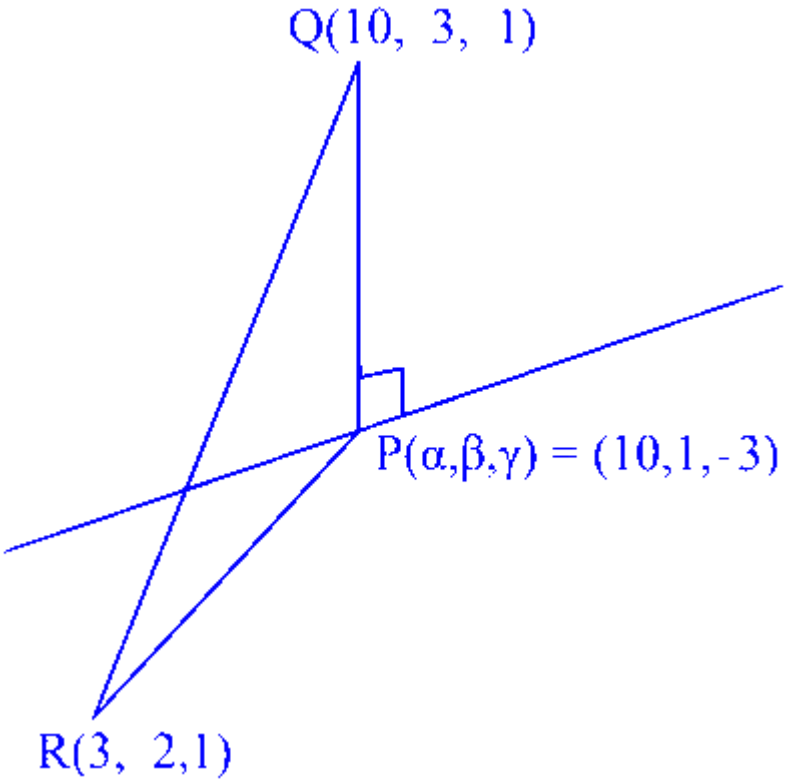
B. $9\sqrt{15}$

C. $3\sqrt{30}$

D. $8\sqrt{15}$

Answer: C

Solution:



$\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}=\lambda$

$\Rightarrow 7\lambda+3,-\lambda+2,-2\lambda-1$

dr's of QP $\Rightarrow$

$7\lambda-7,-\lambda+5,-2\lambda$

Now

$(7\lambda-7)\cdot 7-(-\lambda+5)+(2\lambda)\cdot 2=0$

$54\lambda-54=0\Rightarrow\lambda=1$

$\therefore \mathrm{P}=(10,1,-3)$

$\overrightarrow{\mathrm{PQ}}=-4\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{PR}}=-7\hat{\mathrm{i}}-3\hat{\mathrm{j}}+4\hat{\mathrm{k}}$

$$\text{Area}=\left|\frac{1}{2}\begin{vmatrix} i & j & k \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{vmatrix}\right|=3\sqrt{30}$$

Question8

The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point $(1,4,0)$ along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is :

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Options:

A. $\sqrt{17}$

B. $\sqrt{13}$

C. $\sqrt{15}$

D. $\sqrt{14}$

Answer: D

Solution:

To find the distance from the line

$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

to the point $(1, 4, 0)$ along the line

$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$,

we first consider a parallel line passing through the point $(1, 4, 0)$. The equation of this parallel line is:

$\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3}$.

Next, we find the point of intersection between the given line and the parallel line, which can be expressed in parameter form as:

$(\lambda + 1, 2\lambda + 4, 3\lambda) = (2t + 2, 3t + 6, 4t + 3)$.

Solving for λ in terms of t, from the first coordinate:

$\lambda = 2t + 1$.

Using the second coordinates, we equate:

$2\lambda + 4 = 3t + 6 \quad \Rightarrow \quad 2(2t + 1) + 4 = 3t + 6$.

Simplifying gives:

$4t + 2 + 4 = 3t + 6 \quad \Rightarrow \quad 4t + 6 = 3t + 6 \quad \Rightarrow \quad t = 0$.

Substituting $t = 0$ into either line equation yields the point of intersection (POI) as:

$(2, 6, 3)$.

The distance from the point $(1, 4, 0)$ to the POI $(2, 6, 3)$ is computed as:

$\sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$.

Question9

If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m+n$ is equal to :

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Options:

A. 14

B. 6

C. 21

D. 9

Answer: D

Solution:

$\vec{a}=(2,1,-3)$

$\vec{b}=(-1,-3,-5)$

$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}$

$=2\hat{i}-\hat{j}$

$\vec{b}-\vec{a}=-3\hat{i}-4\hat{j}-2\hat{k}$

$S_d=\frac{|(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}$

$=\frac{2}{\sqrt{5}}$

$(S_d)^2=\frac{4}{5}$

$m=4, n=5 \Rightarrow m+n=9$

Question10

Let in a $\triangle ABC$, the length of the side AC be 6 , the vertex B be $(1,2,3)$ and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle ABC$ is:

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Options:

A. 42

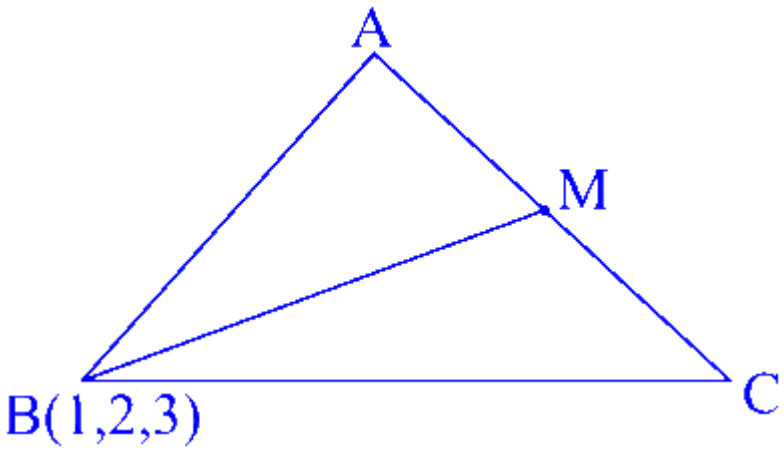
B. 17

C. 56

D. 21

Answer: D

Solution:



$\text{Let M}(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$\overrightarrow{\text{BM}} = (3\lambda+5)\hat{\text{i}} + (2\lambda+5)\hat{\text{j}} + (-2\lambda+4)\hat{\text{k}}$

$\overrightarrow{\text{AC}} \cdot \overrightarrow{\text{BM}} = 0 = 3(3\lambda+5) + 2(2\lambda+5) - 2(-2\lambda+4)$

$\overrightarrow{\text{BM}} = 2\hat{\text{i}} + 3\hat{\text{j}} + 6\hat{\text{k}}$

$|\overrightarrow{\text{BM}} = 7$

$\text{Area } = \frac{1}{2} \times 6 \times 7 = 21$

Question11

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is

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Options:

A. $5\sqrt{6}$

B. 5

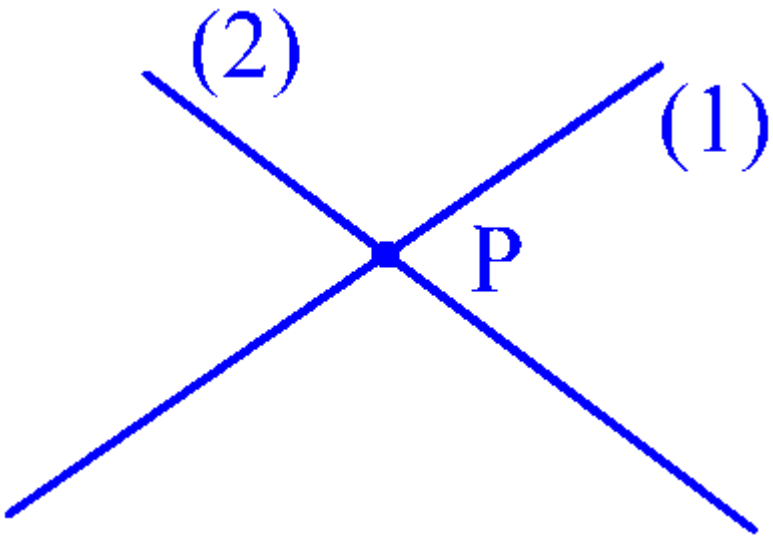
C. $5\sqrt{5}$

D. 10

Answer: C

Solution:

Equation of line through point $(-1, 2, 1)$ is $\rightarrow$



$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} - (2) = \lambda$$

So, $\begin{bmatrix} x = 2\lambda - 1 \\ y = 3\lambda + 2 \\ z = 4\lambda + 1 \end{bmatrix}$

By $(1) \rightarrow \frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}=\mu(\text{ Let })$

So, $\begin{bmatrix} x=3\mu-2 \\ y=2\mu+3 \\ z=\mu+4 \end{bmatrix}$

For intersection point 'P'

$\mathrm{x}=2\lambda-1=3\mu-2$

$\mathrm{y}=3\lambda+2=2\mu+3 \quad \begin{bmatrix} \lambda=1 \\ \mu=1 \end{bmatrix}$

$\mathrm{z}=4\lambda+1=\mu+4$

So, point $\mathrm{P}(\mathrm{x},\mathrm{y},\mathrm{z})=(1,5,5)$

$\&\mathrm{Q}(4,-5,1)$

$\therefore \mathrm{PQ}=\sqrt{9+100+16}$

$=\sqrt{125}=5\sqrt{5}$

Question12

If the image of the point $(4,4,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$ is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to

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Options:

A. 12

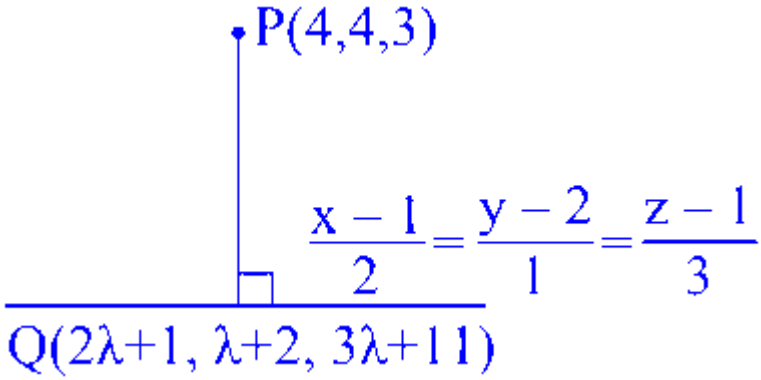
B. 9

C. 7

D. 8

Answer: B

Solution:



$\overrightarrow{PQ} \perp (2\hat{i} + \hat{j} + 3\hat{k})$
$\Rightarrow 2(2\lambda - 3) + 1(\lambda - 2) + 3(3\lambda - 2) = 0$
$\Rightarrow 14\lambda - 14 = 0, \lambda = 1$
So, $Q(3, 3, 4)$
Let image in $R(\alpha, \beta, \gamma)$
$\frac{\alpha + 4}{2} = 3, \frac{\beta + 4}{2} = 3, \frac{\gamma + 3}{2} = 4$
$(\alpha, \beta, \gamma) = (2, 2, 5)$
$\Rightarrow \alpha + \beta + \gamma = 9$

Question13

Let $\mathrm{A}(x, y, z)$ be a point in xy-plane, which is equidistant from three points $(0, 3, 2), (2, 0, 3)$ and $(0, 0, 1)$.

Let $\mathrm{B} = (1, 4, -1)$ and $\mathrm{C} = (2, 0, -2)$. Then among the statements

(S1) : $\triangle\mathrm{ABC}$ is an isosceles right angled triangle, and

(S2) : the area of $\triangle\mathrm{ABC}$ is $\frac{9\sqrt{2}}{2}$,

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Options:

A. both are false

B. only (S2) is true

C. only (S1) is true

D. both are true

Answer: C

Solution:

$A(x, y, z)$ Let $P(0,3,2), Q(2,0,3), R(0,0,1)$
$AP = AQ = AR$
$x^2 + (y-3)^2 + (z-2)^2 = (x-2)^2 + y^2 + (z-3)^2 = x^2 +$
$y^2 + (z-1)^2$

In xy plane z $= 0$

So, $x^2 - 4x + 4 + y^2 + 9 = x^2 + y^2 + 1$

$\Rightarrow y = 2$
$x = 3$
$9 + y^2 - 6y + 9 + 4 = x^2 + y^2 + 1$

So, $A(3,2,0)$ also $B(1,4,-1) \& C(2,0,-2)$

Now $AB = \sqrt{4+4+1} = 3$

$\mathrm{AC} = \sqrt{1+4+4} = 3$
$\mathrm{BC} = \sqrt{1+16+1} = \sqrt{18}$
$\mathrm{AB} = \mathrm{AC}$
isosceles $\Delta \& \mathrm{AB}^2 + \mathrm{AC}^2 = \mathrm{BC}^2$
right angle Δ
Area of $\triangle \mathrm{ABC} = \frac{1}{2} \times$ base.height
$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$
So only S_1 is true

Question14

The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ from the line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ in the direction of the vector $\hat{i} + 4\hat{j} + 7\hat{k}$ is:

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Options:

A.

66

B.

54

C.

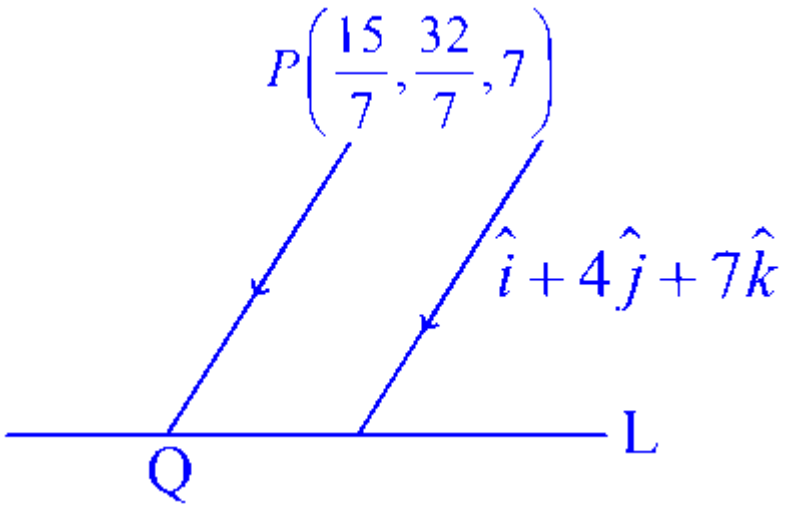
41

D.

44

Answer: A

Solution:



$L = \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

$PQ = \frac{x-\frac{15}{7}}{1} = \frac{y-\frac{32}{7}}{4} = \frac{z-7}{7} = \lambda$

$\Rightarrow \mathrm{Q}\left(\lambda + \frac{15}{7}, 4\lambda + \frac{32}{7}, 7\lambda + 7\right)$

Since Q lies on line L

So, $\frac{\lambda + \frac{15}{7} + 1}{3} = \frac{7\lambda + 7 + 5}{7}$

$\Rightarrow 7\lambda + 22 = 21\lambda + 36$

$\Rightarrow \lambda = -1$

$\therefore$ Point $Q\left(\frac{8}{7}, \frac{4}{7}, 0\right)$

$\mathrm{PQ} = \sqrt{\left(\frac{15}{7} - \frac{8}{7}\right)^2 + \left(\frac{32}{7} - \frac{4}{7}\right)^2 + (7-0)}$

$\mathrm{PQ} = \sqrt{66}$

$\Rightarrow (\mathrm{PQ})^2 = 66$

Question15

Let $\mathrm{L}_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and $\mathrm{L}_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals :

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Options:

A.

25

B.

20

C.

16

D.

18

Answer: A

Solution:

Step 1. Identify the Given Lines

The line

$\mathrm{L}_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$

passes through the point

$P_1 = (1, 2, 1)$

with direction vector

$\mathbf{u} = (1, -1, 2)$.

Similarly, the line

$\mathrm{L}_2: \ \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$

passes through the point

$P_2 = (-1, 2, 0)$

with direction vector

$\mathbf{v} = (-1, 2, 1)$.

Step 2. Determine the Direction of L_3

Since L_3 is perpendicular to both L_1 and L_2, its direction vector must be parallel to the cross product of $\mathbf{u}$ and $\mathbf{v}$. Compute:

$$\mathbf{d} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix}.$$

Using the determinant, we obtain:

$$d_x = (-1)(1) - (2)(2) = -1 - 4 = -5,$$
$$d_y = (2)(-1) - (1)(1) = -2 - 1 = -3,$$
$$d_z = (1)(2) - (-1)(-1) = 2 - 1 = 1.$$

Thus, a valid direction vector is

$\mathbf{d} = (-5, -3, 1)$.

Step 3. Express L_3 in Terms of Its Intersection with L_1

Since L_3 intersects L_1, let the intersection point (on L_1) be expressed using a parameter t as:

$A(t) = (1 + t,\ 2 - t,\ 1 + 2t)$.

Since L_3 passes through $A(t)$ and also through the given point

$Q = (\alpha, \beta, \gamma)$,

its parametric form can be written as:

$(\alpha, \beta, \gamma) = A(t) + s\,\mathbf{d} = (1 + t,\ 2 - t,\ 1 + 2t) + s(-5, -3, 1)$

for some parameters s and t.

Step 4. Express α, β, γ in Terms of t and s

Comparing coordinates, we have:

$$\begin{cases} \alpha = 1 + t - 5s, \\ \beta = 2 - t - 3s, \\ \gamma = 1 + 2t + s. \end{cases}$$

Step 5. Compute $5\alpha - 11\beta - 8\gamma$

Substitute the expressions for α, β, and γ:

$$\begin{aligned}5\alpha - 11\beta - 8\gamma &= 5(1 + t - 5s) - 11(2 - t - 3s) - 8(1 + 2t + s) \\ &= (5 + 5t - 25s) - (22 - 11t - 33s) - (8 + 16t + 8s) \\ &= (5 - 22 - 8) + (5t + 11t - 16t) + (-25s + 33s - 8s) \\ &= -25 + 0t + 0s \\ &= -25.\end{aligned}$$

Taking the absolute value yields:

$|5\alpha - 11\beta - 8\gamma| = 25.$

Conclusion

The value of

$|5\,\alpha - 11\,\beta - 8\,\gamma|$ is $\boxed{25}$.

Question16

Let P be the foot of the perpendicular from the point $(1, 2, 2)$ on the line $\mathrm{L}: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$. Let the line $\vec{r} = (-\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}), \lambda \in \mathbf{R}$, intersect the line L at Q . Then $2(\mathrm{PQ})^2$ is equal to :

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Options:

A.

25

B.

27

C.

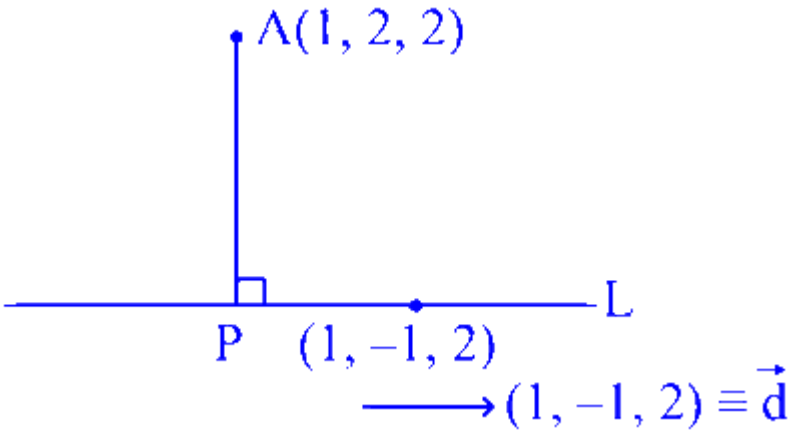
19

D.

29

Answer: B

Solution:



$$L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2} = \mu$$

$$P(\mu+1, -\mu-1, 2\mu+2)$$

$$\overrightarrow{AP} \cdot \vec{d} = 0 \Rightarrow (\mu, -\mu-3, 2\mu) \cdot (1, -1, 2) = 0$$

$$\Rightarrow \mu + \mu + 3 + 4\mu = 0 \Rightarrow \mu = -\frac{1}{2}$$

$$\therefore P\left(\frac{-1}{2}+1, +\frac{1}{2}-1, 2\left(\frac{-1}{2}\right)+2\right)$$

$$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$$

$$\mu + 1 = -1 + \lambda \quad \Big| \quad -\mu - 1 = 1 - \lambda \quad \Big| \quad 2\mu + 2 = -2 + \lambda$$
$$\mu = \lambda - 2 \quad \Big| \quad \mu = \lambda - 2 \quad \Big| \quad \downarrow$$

$$2(\lambda - 2) + 2 = -2 + \lambda$$
$$2\lambda - 4 + 2 = -2 + \lambda$$

$$\therefore \quad \mu = -2, \lambda = 0$$

$$\therefore \quad Q \equiv (-1, 1-2)$$

$$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right) \text{ and } Q(-1, 1, -2)$$

$$PQ = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{-1}{2}-1\right)^2 + (1+2)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{9}{4} + 9} = \sqrt{\frac{54}{4}}$$

$$\therefore \quad 2(PQ)^2 = 2\left(\frac{54}{4}\right) = 27$$

Question17

Let a straight line L pass through the point $P(2, -1, 3)$ and be perpendicular to the lines $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$ and $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is:

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Options:

A.

$\sqrt{10}$

B.

2

C.

$2\sqrt{3}$

D.

3

Answer: D

Solution:

To find the distance between the points P and Q, let's consider the following steps:

Determine the direction vector of line L:

For the line L to be perpendicular to both given lines, we find the direction vector of L using the cross product of the direction vectors of the given lines.

The first line has direction vector $\langle 2, 1, -2 \rangle$ and the second line has direction vector $\langle 1, 3, 4 \rangle$. The cross product is:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 10\hat{i} - 10\hat{j} + 5\hat{k} = 5(2\hat{i} - 2\hat{j} + \hat{k})$$

Thus, the direction vector of line L is $\langle 2, -2, 1 \rangle$.

Equation of line L:

Since L passes through point $P(2, -1, 3)$ with direction vector $\langle 2, -2, 1 \rangle$, its parametric form is:

$\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-3}{1} = \lambda$

Find the intersection with the yz-plane:

On the yz-plane, $x = 0$. Substitute into the line equation to find λ:

$2\lambda + 2 = 0 \implies \lambda = -1$

Substitute $\lambda = -1$ back into the parametric equations to find Q:

$x = 2(-1) + 2 = 0, \quad y = -2(-1) - 1 = 1, \quad z = (-1) + 3 = 2$

Therefore, the intersection point Q is $(0, 1, 2)$.

Calculate the distance between P and Q:

$\mathrm{d}(P, Q) = \sqrt{(2-0)^2 + (-1-1)^2 + (3-2)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

Thus, the distance between points P and Q is 3.

Question18

Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5,6 and 7 square units respectively. Then the area (in square units) of the $\triangle BCD$ is equal to :

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Options:

A. $\sqrt{110}$

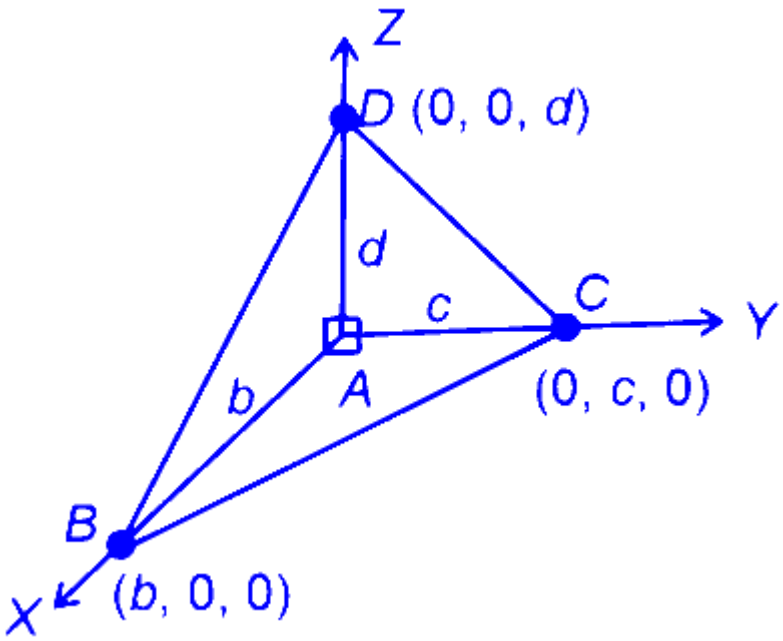
B. 12

C. $\sqrt{340}$

D. $7\sqrt{3}$

Answer: A

Solution:



$\text{ar}(\triangle ABC) = 5$

$\frac{1}{2} \times bc = 5$

$\Rightarrow \quad bc = 10$

$\text{ar}(\triangle ACD) = 6$

$\frac{1}{2} \times cd = 6$

$\Rightarrow \quad cd = 12$

$\text{ar}(\triangle ABD) = 7$

$bd = 14$

$\text{area}(\triangle BCD) = \frac{1}{2}|\overrightarrow{BC} \times \overrightarrow{BD}|$

$\overrightarrow{BC} =< -b, c, 0 >$

$\overrightarrow{BD} =< -b, 0, d >$

$|\overrightarrow{BC} \times \overrightarrow{BD}| = \sqrt{c^2d^2 + b^2d^2 + b^2c^2}$

$= \sqrt{12^2 + 14^2 + 10^2}$

$= \sqrt{440}$

$\text{ar}(\triangle BCD) = \sqrt{110}$

Question19

Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, $\text{QR} = 5$ and the coordinates of the point P be $(0, 2, 3)$. If the area of the triangle PQR is $\frac{m}{n}$ then :

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Options:

A. $2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

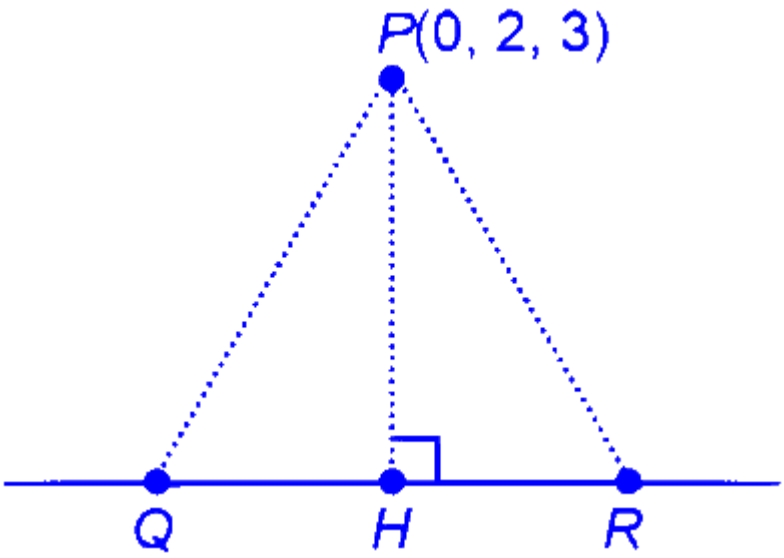
B. $\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

C. $5\,\mathrm{m} - 21\sqrt{2}\mathrm{n} = 0$

D. $5\,\mathrm{m} - 2\sqrt{21}\mathrm{n} = 0$

Answer: A

Solution:



$H : (5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$
$< \text{DR of PH} >$
$< 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 >$
$\overrightarrow{PH} \cdot \overrightarrow{QR} = 0$
$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$
$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$
$\Rightarrow 38\lambda = 38$
$\Rightarrow \lambda = 1$
$\therefore \quad H(2, 3, -1)$
$PH = \sqrt{4 + 1 + 16} = \sqrt{21}$
$\therefore \quad \text{Area } = \frac{1}{2} \times PH \text{ QR}$
$= \frac{1}{2} \times \sqrt{21} \times 5 = \frac{5\sqrt{21}}{2} = \frac{m}{n}$
$2m - 5\sqrt{21}n = 0$

Question20

The line L_1 is parallel to the vector $\overrightarrow{\mathrm{a}} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$ and the line L_2 is parallel to the vector $\overrightarrow{\mathrm{b}} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is :

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Options:

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{38}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{57}}$

Answer: A

Solution:

The line L_1 is parallel to the vector $\overrightarrow{\mathrm{a}} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$. The line L_2 is parallel to the vector $\overrightarrow{\mathrm{b}} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$.

Equations of the Lines:

Equation of L_1:

$\mathbf{r} = 7\hat{i} + 6\hat{j} + 2\hat{k} + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Equation of L_2:

$\mathbf{r} = 5\hat{i} + 3\hat{j} + 4\hat{k} + \mu(2\hat{i} + \hat{j} + 3\hat{k})$

Shortest Distance Between L_1 and L_2:

The formula for the shortest distance between two skew lines is given by:

$\text{Distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|} \right|$

Calculations:

Vector between the points:

$\vec{a}_2 - \vec{a}_1 = (5\hat{i} + 3\hat{j} + 4\hat{k}) - (7\hat{i} + 6\hat{j} + 2\hat{k}) = -2\hat{i} - 3\hat{j} + 2\hat{k}$

Cross product of direction vectors:

$\vec{b}_1 = -3\hat{i} + 2\hat{j} + 4\hat{k}, \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} + 17\hat{j} - 7\hat{k}$

Magnitude of the cross product:

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{2^2 + 17^2 + (-7)^2} = \sqrt{342}$

Dot product:

$\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) = (-2\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})$

$= (-2)(2) + (-3)(17) + (2)(-7) = -4 - 51 - 14 = -69$

Shortest Distance:

$\text{Distance} = \left| \frac{-69}{\sqrt{342}} \right| = \frac{23}{\sqrt{38}}$

Question21

If the image of the point $\mathrm{P}(1,0,3)$ in the line joining the points $\mathrm{A}(4,7,1)$ and $\mathrm{B}(3,5,3)$ is $Q(\alpha,\beta,\gamma)$, then $\alpha+\beta+\gamma$ is equal to :

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Options:

A. $\frac{46}{3}$

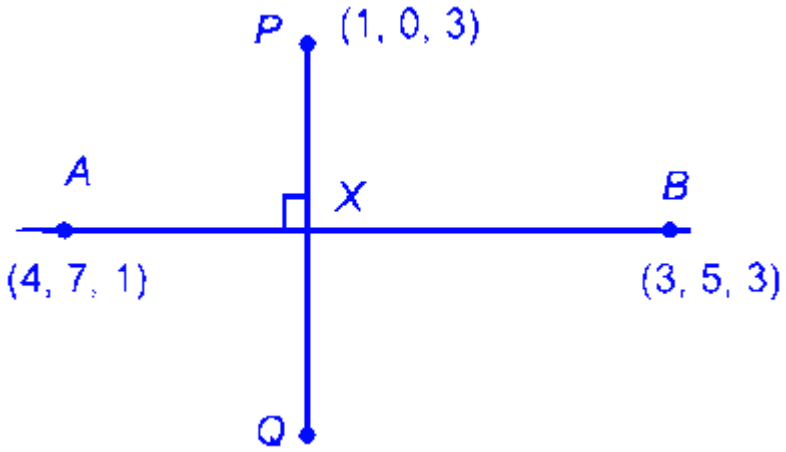
B. 18

C. 13

D. $\frac{47}{3}$

Answer: A

Solution:



Let X be mid point of P and Q, which would be also feet of perpendicular.

Let X divides A and B in $\lambda:1, \lambda \neq -1$

$$X=\left(\frac{3\lambda+4}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}, \frac{3\lambda+1}{\lambda+1}\right)$$

Now $PX \perp AB \Rightarrow \overrightarrow{PX} \cdot \overrightarrow{AB}=0$

$$\left(\frac{3\lambda+4}{\lambda+1}-1\right) \cdot (4-3)+\left(\frac{5\lambda+7}{\lambda+1}-0\right)(7-5)$$

$$+\left(\frac{3\lambda+1}{\lambda+1}-3\right) \cdot (1-3)=0$$

$$\frac{2\lambda+3}{\lambda+1}+\frac{10\lambda+14}{\lambda+1}+\frac{4}{\lambda+1}=0$$

$$\Rightarrow \frac{12\lambda+21}{\lambda+1}=0 \Rightarrow \lambda=\frac{-7}{4}$$

$$X=\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$$

X is mid point of PQ

$$Q\equiv\left(2\cdot\frac{5}{3}-1,2\cdot\frac{7}{3}-0,2\cdot\frac{17}{3}-3\right)\equiv(\alpha,\beta,\gamma)$$

$$\Rightarrow \quad \alpha+\beta+\gamma=\frac{2(5+7+17)}{3}-4=\frac{58}{3}-4=\frac{46}{3}$$

Question22

Line L_1 passes through the point $(1,2,3)$ and is parallel to z-axis. Line L_2 passes through the point $(\lambda,5,6)$ and is parallel to y-axis. Let for $\lambda=\lambda_1,\lambda_2,\lambda_2<\lambda_1$, the shortest distance between the two lines be 3 . Then the square of the distance of the point $(\lambda_1,\lambda_2,7)$ from the line L_1 is

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Options:

A. 25

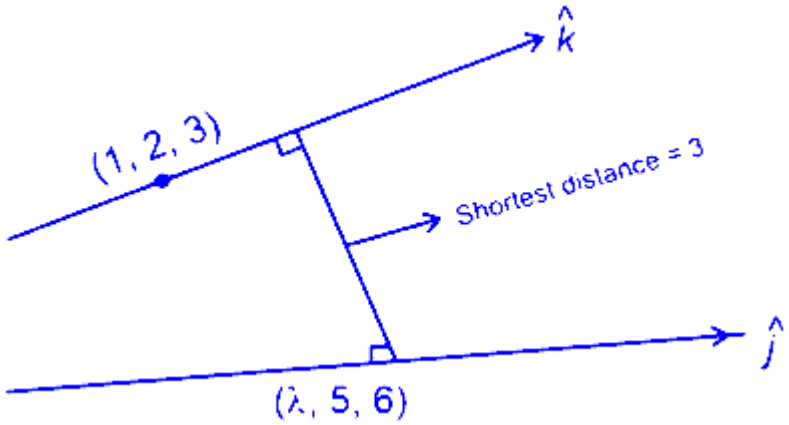
B. 32

C. 40

D. 37

Answer: A

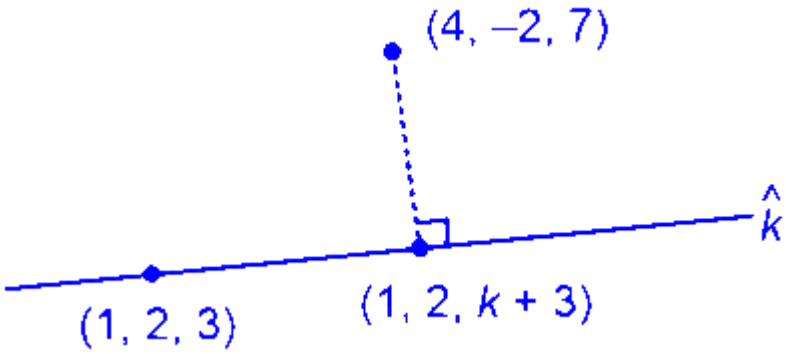
Solution:



$3 = \frac{|[(\lambda-1)\hat{i}+3\hat{j}+3\hat{k}].(\hat{k}\times\hat{j})|}{|\hat{k}\times\hat{j}|} = |\lambda-1| = 3$

$\Rightarrow \lambda = 4 \text{ or } -2, \lambda_2 < \lambda_1 \Rightarrow \lambda_2 = -2$

$\lambda_1 = 4$



$(3, -4, 4-k) \perp \hat{k}$

$4 - k = 0 \Rightarrow k = 4$

$\Rightarrow$ Distance between $(4,-2,7)\&(1,2,7)$

$\Rightarrow \sqrt{3^2+4^2} = 5$

Question23

Let a line passing through the point $(4,1,0)$ intersect the line $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $A(\alpha,\beta,\gamma)$ and the line $L_2 : x-6 = y = -z+4$ at the point $B(a,b,c)$. Then $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to

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Options:

A. 16

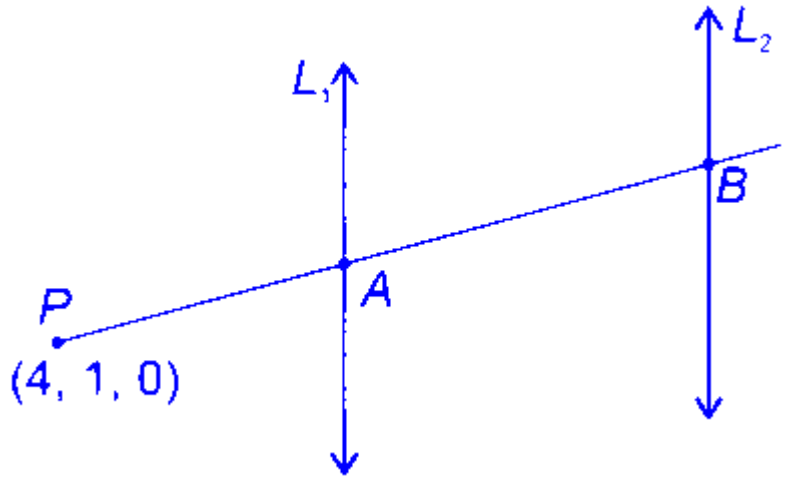
B. 6

C. 8

D. 12

Answer: C

Solution:



$A = (\alpha, \beta, 4) = (2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$
and $B = (a, b, c) = (\mu + 6, \mu, -\mu + 4)$
$\because \dfrac{\mu + 2}{2\lambda - 3} = \dfrac{\mu - 1}{3\lambda + 1} = \dfrac{-\mu + 4}{4\lambda + 3}$
$\therefore \quad \lambda\mu + 8\lambda + 4\mu = 1 \quad \ldots(1)$
and $7\lambda\mu - 16\lambda + 4\mu = 7 \quad \ldots(2)$

from equation (1) and (2)

$\lambda = -1$ and $\mu = 3$

Or $\lambda = -\frac{1}{3}$ and $\mu = 1$

By taking, $\lambda = -1$ and $\mu = 3$, we get

$\therefore \quad A(\alpha, \beta, 4) = (-1, -1, -1)$
and $B(a, b, c) = (9, 3, 1)$

$$\therefore \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 8$$

Question24

The distance of the point $(7, 10, 11)$ from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

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Options:

A. 16

B. 12

C. 18

D. 14

Answer: D

Solution:

Equation of line passing through $P(7, 10, 11)$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

$\frac{x-7}{2} = \frac{y-10}{3} = \frac{z-11}{6} = \lambda$

Let the point on the line is

$Q(2\lambda + 7, 3\lambda + 10, 6\lambda + 11)$

Q lies on line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$

$3\lambda + 10 = 4 \Rightarrow \lambda = -2$
$\therefore \quad Q(3, 4, -1)$
$PQ = \sqrt{16 + 36 + 144} = 14$

Question25

Each of the angles β and γ that a given line makes with the positive y - and z-axes, respectively, is half of the angle that this line makes with the positive x-axes. Then the sum of all possible values of the angle β is

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Options:

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{3\pi}{2}$

Answer: C

Solution:

Given:

Each of the angles β and γ is half of the angle that the line makes with the positive x-axis, i.e., $\beta = \gamma = \frac{\alpha}{2}$.

The equation for the direction cosines of angles a line makes with the coordinate axes is given by:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Since $\beta = \gamma$, we substitute to get:

$\cos^2\alpha + 2\cos^2\beta = 1$

Substitute $\cos\beta = \cos\gamma$:

$\cos\alpha = 2\cos^2\beta - 1$

Replacing back into the equation:

$(2\cos^2\beta - 1)^2 + 2\cos^2\beta = 1$

Simplify:

$(2\cos^2\beta - 1)(2\cos^2\beta + 1) = 0$

Solving for $\cos^2\beta$, we have:

$2\cos^2\beta - 1 = 0 \quad \text{or} \quad 2\cos^2\beta + 1 = 0$

The latter gives no real solutions, thus:

$\cos^2\beta = \frac{1}{2}$

Therefore, $\beta = \frac{\pi}{4}$ or $\beta = \frac{\pi}{2}$.

Thus, the sum of all possible values of β is:

$\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

Question26

Let A and B be two distinct points on the line $L: \frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1,2,3)$ on the line L. If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to

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Options:

A. 49

B. 21

C. 47

D. 62

Answer: C

Solution:

$L: \frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$

Point $A(3\lambda+6, 2\lambda+7, 7-2\lambda)$

$B(3\mu+6, 2\mu+7, 7-2\mu)$

Let $P(3k+6, 2k+7, 7-2k)$ be foot of perpendicular from $P'(1,2,3)$

$\therefore \quad PP' \langle 3,2,-2 \rangle = 0$

$3(3k+5)+(2k+5)2+(4-2k)(-2)=0$

$9k+15+4k+10-8+4k=0$

$17k+17=0$

$\Rightarrow k=-1 \qquad \therefore \quad P(3,5,9)$

$|\overrightarrow{AP}|=2\sqrt{17}$

$\Rightarrow (3\lambda+3)^2+(2\lambda+2)^2+(-2-2\lambda)^2=17\times 4$
$=17(\lambda+1)^2=17\times 4$
$\Rightarrow \lambda+1=\pm 2 \Rightarrow \lambda=1 \text{ or } \lambda=-3$
$\therefore A(9,9,5), B(-3,1,13)$
$\overrightarrow{OA}\cdot\overrightarrow{OB}=(9\hat{i}+9\hat{j}+5\hat{k})\cdot(-3\hat{i}+\hat{j}+13\hat{k})$
$=-27+9+65=47$

Question27

Let the shortest distance between the lines $\frac{x-3}{3}=\frac{y-\alpha}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha+\beta$ is

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Options:

A. 42

B. 40

C. 48

D. 46

Answer: D

Solution:

$L_1: \frac{x-3}{3}=\frac{y-\alpha}{-1}=\frac{z-3}{1}$
$L_2: \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-\beta}{4}$

$a_1:(3,\alpha,3) \quad a^2=(-3,-7,\beta)$
$\vec{b}_1=3\hat{i}-\hat{j}+\hat{k} \quad \vec{b}_2=-3\hat{i}+2\hat{j}+4\hat{k}$

$d=\frac{\left|(\vec{a}_2-\vec{a}_1)\cdot\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}=3\sqrt{30}$

$=\frac{\begin{vmatrix}6 & \alpha+7 & 3-\beta\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}}{\sqrt{6^2+15^2+3^2}}=3\sqrt{30}$

$\Rightarrow|-15\alpha-3\beta-132|=270$

$|5\alpha+\beta+44|=90$

$\Rightarrow 5\alpha+\beta=90-44=46$

Question28

Let the values of p , for which the shortest distance between the lines $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$ and $\overrightarrow{\mathrm{r}}=(\mathrm{p}\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be $\mathrm{a}, \mathrm{b}, (\mathrm{a}<\mathrm{b})$. Then the length of the latus rectum of the ellipse $\frac{x^2}{\mathrm{a}^2}+\frac{y^2}{\mathrm{b}^2}=1$ is :

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Options:

A. $\frac{3}{2}$

B. 9

C. 18

D. $\frac{2}{3}$

Answer: D

Solution:

$\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}; (p\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

$d=\left|\frac{(\vec{a}-\vec{b})\cdot(\vec{p}_1\times\vec{p}_2)}{|\vec{p}_1\times\vec{p}_2|}\right|=\frac{1}{\sqrt{6}}$

$\vec{a}-\vec{b}=(p+1)\hat{i}+2\hat{j}+\hat{k}$

$\vec{p}_1\times\vec{p}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\2&3&4\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$

$\frac{1}{\sqrt{6}}=\left|\frac{(p+1)-4+1}{\sqrt{6}}\right|$

$=|p-2|=1\Rightarrow p=3,1$

$a=1, b=3$

$\frac{x^2}{1}+\frac{y^2}{9}=1$

Length of $LR=\frac{2a^2}{b}=\frac{2}{3}$

Question29

Let A be the point of intersection of the lines $\mathrm{L}_1: \frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$ and $\mathrm{L}_2: \frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB=AC=\sqrt{15}$. Then the square of the area of the triangle ABC is :

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Options:

A. 63

B. 57

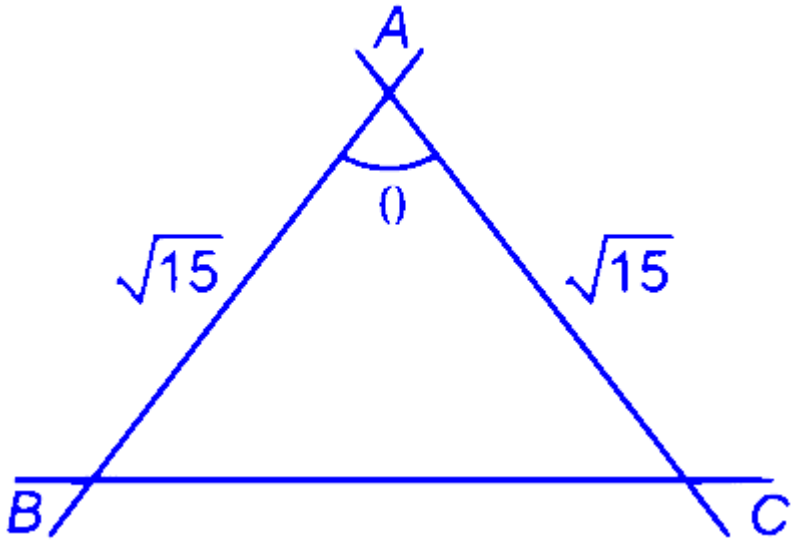
C. 60

D. 54

Answer: D

Solution:

$L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}; L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$



$$\cos\theta = \left|\frac{3+0-5}{\sqrt{2}\times\sqrt{50}}\right|$$
$$= \frac{2}{10} = \frac{1}{5}$$
$$\therefore \sin\theta = \frac{2\sqrt{6}}{5}$$
$$\text{Area } = \frac{1}{2}ab\sin\theta$$
$$= \frac{1}{2}\times\sqrt{15}\times\sqrt{15}\times\frac{2\sqrt{6}}{5}$$
$$= 3\sqrt{6}$$
$$(\text{ Area })^2 = 9\times 6 = 54$$

Question30

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

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Options:

A. $\frac{3}{2}$

B. 3

C. -3

D. $-\frac{3}{2}$

Answer: C

Solution:

$$d=\left|\frac{(\vec{a}-\vec{b})\cdot\vec{p}_1\times\vec{p}_2}{|\vec{p}_1\times\vec{p}_2|}\right|=\frac{5}{\sqrt{6}}$$

$$\vec{a}-\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$$

$$\overrightarrow{p_1}\times\overrightarrow{p_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\1&\alpha&1\end{vmatrix}$$

$$=\hat{i}(3-4\alpha)-\hat{j}(-2)+\hat{k}(2\alpha-3)$$

$$(\vec{a}-\vec{b})\cdot\left|\overrightarrow{p_1}\times\overrightarrow{p_2}\right|=3-4\alpha+4-4\alpha+6$$

$$=13-8\alpha$$

$$\left|\frac{13-8\alpha}{\sqrt{(3-4\alpha)^2+4+(2\alpha-3)^2}}\right|=\frac{5}{\sqrt{6}}$$

$$\left|\frac{13-8\alpha}{\sqrt{20\alpha^2-36\alpha+22}}\right|=\frac{5}{\sqrt{6}}$$

$$=6(13-8\alpha)^2=25\left(20\alpha^2-36\alpha+22\right)$$

$$=116\alpha^2+348\alpha-464=0$$

Sum of roots $=-3$

Question31

Let the line L pass through $(1,1,1)$ and intersect the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1}$. Then, which of the following points lies on the line L ?

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Options:

A. $(7,15,13)$

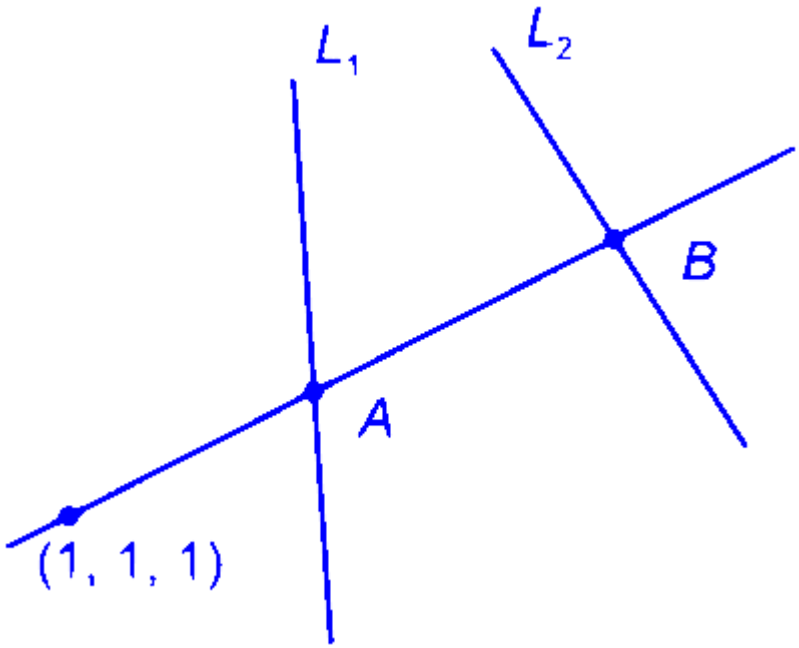
B. $(4,22,7)$

C. $(10, -29, -50)$

D. $(5, 4, 3)$

Answer: A

Solution:



$L : \dfrac{x-1}{a} = \dfrac{y-1}{b} = \dfrac{z-1}{c}$

$L_1 : \dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4} = \lambda$ (say)

Any point on L_1 be $A(2\lambda+1, 3\lambda+1, 4\lambda+1)$

$L_2 : \frac{x-3}{1} = \frac{y-4}{2} = \frac{z}{1} = \mu$ (say)

Any point on L_2 be $B(\mu+3, 2\mu-4, \mu)$

D of L be : $< 2\lambda, 3\lambda-2, 4\lambda >$ or $< \mu+2, 2\mu+3, \mu-1 >$

Now $\frac{2\lambda}{\mu+2} = \frac{3\lambda-7}{2\mu+3} = \frac{4\lambda}{\mu-1}$

$\Rightarrow \lambda = \dfrac{-6}{5} \quad \mu = -5$

$\therefore < a, b, c > \equiv \langle -3, -7, -6 > \text{ or } < 3, 7, 6 >$

$\therefore \quad L : \dfrac{x-1}{3} = \dfrac{y-1}{7} = \dfrac{z-1}{6}$

$(7, 15, 13)$ lies on the line.

Question32

Consider the lines L_1: x - 1 = y - 2 = z and L_2: x - 2 = y = z - 1. Let the feet of the perpendiculars from the point P(5, 1, -3) on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A, then $4A^2$ is equal to :

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Options:

A.

151

B.

147

C.

139

D.

143

Answer: B

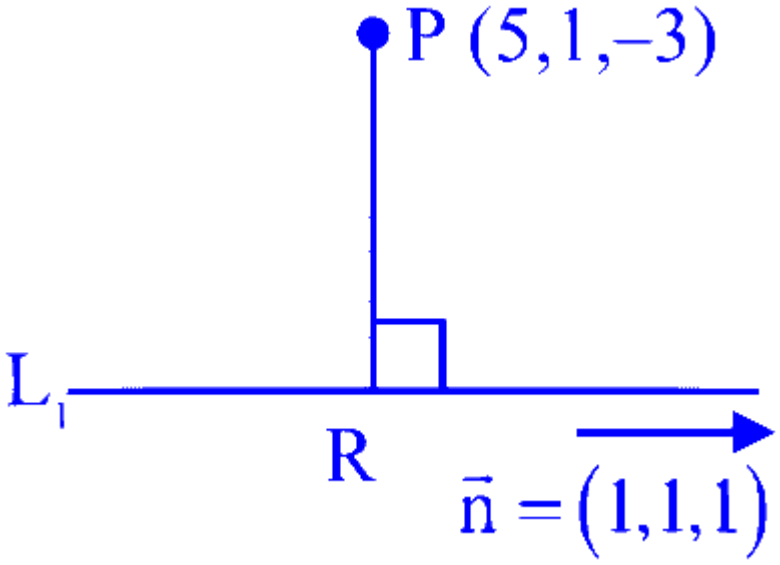
Solution:

$$L_1 : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-0}{2}$$

Let $Q(\lambda+1, \lambda+2, \lambda)$

$$\overrightarrow{PQ} = (\lambda-4, \lambda-1, \lambda+3)$$

$$\overrightarrow{PQ} \cdot \vec{m} = 0$$



$\Rightarrow \lambda-4+\lambda+1, \lambda+3=0$
$\Rightarrow 3\lambda=0$
$\Rightarrow \lambda=0$
$\Rightarrow \mathrm{Q}(1,2,0)$
$\mathrm{L}_2: \dfrac{\mathrm{x}-2}{1}=\dfrac{\mathrm{y}-0}{1}=\dfrac{\mathrm{z}-1}{2}$
Let $\mathrm{R}(\mu+2, \mu, \mu+1) \overrightarrow{\mathrm{PR}}=(\mu-3, \mu-1, \mu+4)$
$\overrightarrow{\mathrm{PR}} \cdot \vec{\mathrm{n}}=0$
$\mu-3+\mu-1+\mu+4=0$
$\neq \mu=0$
$\mathrm{R}(2,0,1)$

Area of $\triangle \mathrm{PQR}(\mathrm{A})=\dfrac{1}{2}|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}|$
$\mathrm{A}=\dfrac{1}{2}|(-4\hat{\mathrm{i}}+\hat{\mathrm{j}}+3\hat{\mathrm{k}}) \times(-3\hat{\mathrm{i}}+\hat{\mathrm{j}}+4\hat{\mathrm{k}})|$
$\mathrm{A}=\dfrac{1}{2}|7(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}})|$
$\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ -4 & 1 & 3 \\ -3 & -1 & 4\end{vmatrix}$
$=7\hat{\mathrm{i}}+7\hat{\mathrm{j}}+7\hat{\mathrm{k}}$
$4\ \mathrm{A}^2=49 \times 3=147$

Question33

If the equation of the line passing through the point $\left(0,-\frac{1}{2}, 0\right)$ and perpendicular to the lines $\vec{r}=\lambda\left(\hat{i}+a \hat{j}+b \hat{k}\right)$ and

$\vec{r}=\left(\hat{i}-\hat{j}-6\hat{k}\right)+\mu\left(-b\hat{i}+a\hat{j}+5\hat{k}\right)$ is $\frac{x-1}{-2}=\frac{y+4}{d}=\frac{z-c}{-4}$, then $a+b+c+d$ is equal to :

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Options:

A.

13

B.

14

C.

12

D.

10

Answer: B

Solution:

The line we need to find should be orthogonal to two given lines, meaning it will be parallel to the cross product of their direction vectors:

$(\hat{i}+a\hat{j}+b\hat{k})\times(-b\hat{i}+a\hat{j}+5\hat{k})$

Calculating the cross product results in the vector:

$\hat{i}(5a-ab)-\hat{j}(b^2+5)+\hat{k}(a+ab)$

This shows that the direction ratios of the required line are proportional to:

$\alpha(5a-ab),-(b^2+5),(a+ab)$

Since the line is also expressed as $\frac{x-1}{-2}=\frac{y+4}{d}=\frac{z-c}{-4}$, the direction ratios based on this equation are:

$\alpha(-2),d,-4$

Thus, we have the equation:

$\frac{5a-ab}{-2}=\frac{-(b^2+5)}{d}=\frac{a+ab}{-4}$

The point $\left(0, -\frac{1}{2}, 0\right)$ lies on the line, which implies:

$\frac{0-1}{-2} = \frac{-\frac{1}{2}+4}{d} = \frac{0-c}{-4}$

Simplifying these gives $d = 7$ and $c = 2$.

Substitute into the main equation:

$\frac{5a-ab}{-2} = \frac{a+ab}{-4}$

$\frac{-b^2-5}{7} = \frac{a+ab}{-4}$

Solving these yields:

$$\begin{array}{c|c} -20a + 4ab = -2a - 2ab & 4b^2 + 20 = 70 + 7ab \\ 18a = 6ab & 36 + 20 = 70 + 21a \\ b = 3 & 56 = 28a \Rightarrow a = 2 \end{array}$$

Substituting these into the variables, we find:

$a + b + c + d = 2 + 3 + 2 + 7 = 14$

Question34

Let the values of λ for which the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points $(0, 0), (\lambda_1, \lambda_2)$ and (λ_2, λ_1) is

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Options:

A.

3

B.

$\frac{5\sqrt{2}}{3}$

C.

$\frac{\sqrt{2}}{3}$

D.

4

Answer: B

Solution:

$\vec{p}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{q}=3\hat{i}+4\hat{j}+5\hat{k}$

$\Rightarrow \vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$

$A\equiv(1,2,3) B\equiv(\lambda,4,5)$

$\text{Shortest Distance } = \left|\frac{\overrightarrow{AB}\cdot(\vec{P}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$\frac{1}{\sqrt{6}}=\left|\frac{(\lambda-1)\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{6}}\right|$

$\Rightarrow |-\lambda+1+4-2|=1\Rightarrow|\lambda-3|=1$

$\Rightarrow \lambda=3\pm1=4,2$

Radius of circle passing through points

$(0,0),(4,2)\&(2,4)$

$=\frac{abc}{4\Delta}=\frac{\sqrt{20}\times\sqrt{20}\times\sqrt{8}}{4\times\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ 0 & 4 & 2\\ 0 & 2 & 4\end{vmatrix}}=\frac{20\times2\sqrt{2}}{2\times12}$

$=\frac{5\sqrt{2}}{3}$

Question35

If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is : [27-Jan-2024 Shift 1]

Options:

A. 5

B. 8

C. 7

D. 10

Answer: B

Solution:

Solution:

$$\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$$

$$\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$$

the shortest distance between the lines

$$=\left|\frac{(\vec{a}-\vec{b})\cdot(\vec{d_1}\times\vec{d_2})}{|\vec{d_1}\times\vec{d_2}|}\right|$$

$$=\left|\frac{\begin{vmatrix}\lambda-4 & 0 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}\right|$$

$$=\left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{|2\hat{i}-1\hat{j}+0\hat{k}}\right|$$

$$\frac{6}{\sqrt{5}}=\left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$$3=|\lambda-4|$$

$$\lambda-4=\pm3$$

$$\lambda=7,1$$

Sum of all possible values of λ is $=8$

Question36

The distance, of the point (7, −2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :
[27-Jan-2024 Shift 1]

Options:

A. 12

B. 14

C. 18

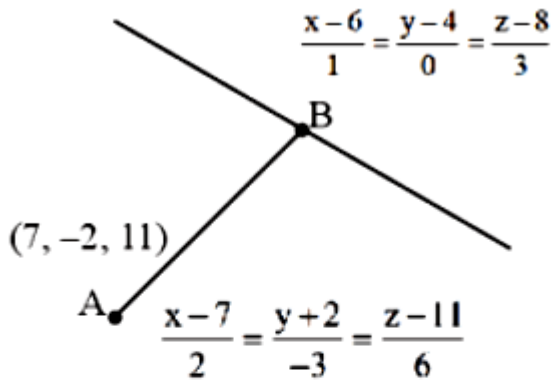
D. 21

Answer: B

Solution:

Solution:

$B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$$

$$-3\lambda - 6 = 0$$

$$\lambda = -2$$

$$B \Rightarrow (3, 4, -1)$$

$$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$$

$$= \sqrt{16 + 36 + 144}$$

$$= \sqrt{196} = 14$$

Question37

The position vectors of the vertices A, B and C of a triangle are $2\hat{i}-3\hat{j}+3\hat{k}$, $2\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+\hat{j}+3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :
[27-Jan-2024 Shift 2]

Options:

A. 49

B. 42

C. 50

D. 45

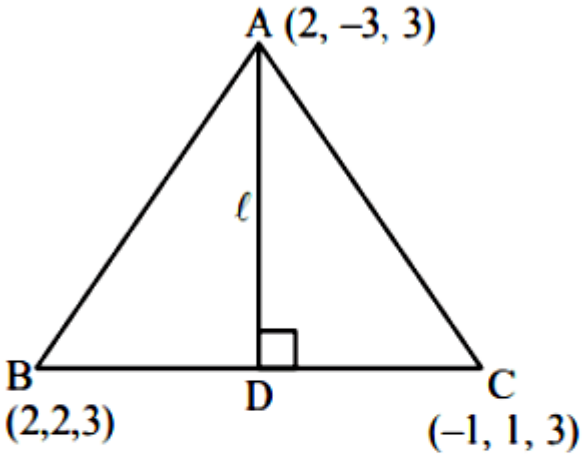
Answer: D

Solution:

Solution:

AB = 5

AC = 5



∴D is midpoint of BC

$D\left(\frac{1}{2}, \frac{3}{2}, 3\right)$

$$\therefore l = \sqrt{\left(2-\frac{1}{2}\right)^2+\left(-3-\frac{3}{2}\right)^2+(3-3)^2}$$

$$l = \sqrt{\frac{45}{2}}$$

$$\therefore 2l^2 = 45$$

Question38

Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2+l_2^2+l_3^2$ equals :
[27-Jan-2024 Shift 2]

Options:

A. $\frac{1}{5}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

Answer: B

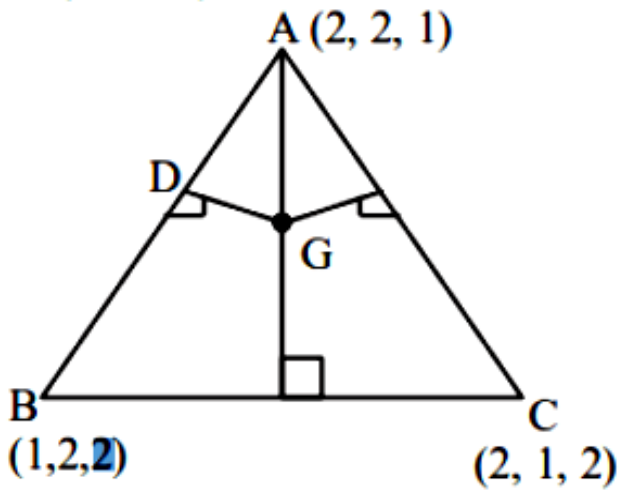
Solution:

Solution:

$\triangle ABC$ is equilateral

Orthocentre and centroid will be same

$G\left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right)$



Mid-point of AB is $D\left(\frac{3}{2}, 2, \frac{3}{2}\right)$

$$\therefore \ell_1 = \sqrt{\frac{1}{36} + \frac{1}{9} + \frac{1}{36}}$$

$$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$$

$$\therefore \ell_1^2 + \ell_2^2 + \ell_3^2 = \frac{1}{2}$$

Question39

Let the image of the point (1, 0, 7) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?
[27-Jan-2024 Shift 2]

Options:

A. $(1, -2, 1+\sqrt{2})$

B. $(1, 2, 1-\sqrt{2})$

C. $(3, 4, 3-2\sqrt{2})$

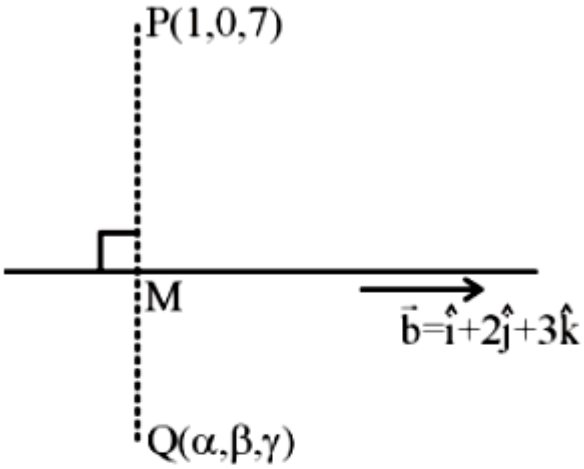
D. $(3, -4, 3+2\sqrt{2})$

Answer: C

Solution:

Solution:

$$L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$



$M(\lambda, 1+2\lambda, 2+3\lambda)$

$$\overrightarrow{PM} = (\lambda-1)\hat{i} + (1+2\lambda)\hat{j} + (3\lambda-5)\hat{k}$$

$\overrightarrow{PM}$ is perpendicular to line L_1

$$\overrightarrow{PM} \cdot \vec{b} = 0 \quad \left(\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$$

$$14\lambda = 14 \Rightarrow \lambda = 1$$

$$\therefore M = (1, 3, 5)$$

$$\vec{Q} = 2\vec{M} - \vec{P} \left[M \text{ is midpoint of } \vec{P} \& \vec{Q}\right]$$

$$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$$

$$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$$

$$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$$

Required line having direction cosine (l, m, n)

$$l^2+m^2+n^2=1$$

$$\Rightarrow l^2+\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{\sqrt{2}}\right)^2=1$$

$$l^2=\frac{1}{4}$$

$\therefore l=\frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through $(1, 6, 3)$ will be

$$\vec{r}=\left(\hat{i}+6\hat{j}+3\hat{k}\right)+\mu\left(\frac{1}{2}\hat{i}-\frac{1}{2}\hat{j}-\frac{1}{\sqrt{2}}\hat{k}\right)$$

Option (3) satisfying for $\mu=4$

Question40

The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to.....
[27-Jan-2024 Shift 2]

Answer: 108

Solution:

Solution:

$$\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$$

$$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$$

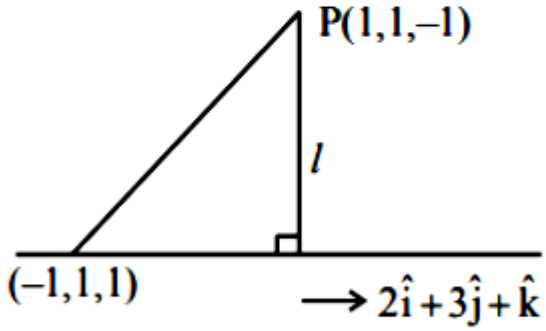
$$\Rightarrow \lambda+2=4k-3$$

$$-\lambda=3k-2$$

$$\Rightarrow k=1, \lambda=-1$$

$$8\lambda+7=k-2$$

$$\therefore P=(1,1,-1)$$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$$

$$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$$

$$\Rightarrow 14l^2=108$$

Question41

Let O be the origin and the position vector of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is
[29-Jan-2024 Shift 1]

Options:

A. $\frac{2}{3}\sqrt{31}$

B. $\frac{2}{3}\sqrt{34}$

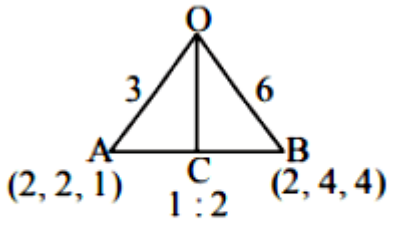
C. $\frac{3}{4}\sqrt{34}$

D. $\frac{3}{2}\sqrt{31}$

Answer: B

Solution:

Solution:



length of $OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

Question42

Let PQR be a triangle with R(−1, 4, 2). Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of △PQR from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is [29-Jan-2024 Shift 1]

Options:

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Answer: C

Solution:

Solution:

Centroid G divides MR in $1:2$

$G(1, 2, 2)$

Point of intersection A of given lines is $(2, -6, 0)$

$AG = \sqrt{69}$

Question43

A line with direction ratios 2, 1, 2 meets the lines x = y + 2 = z and x + 2 = 2y = 2z respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is l , then l^2 is [29-Jan-2024 Shift 1]

Answer: 65

Solution:

Solution:

Let P(t, t − 2, t) and Q(2 s − 2, s, s)

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2}=\frac{s-t+2}{1}=\frac{s-t}{2}$$

$\Rightarrow t=6$ and $s=2$

$\Rightarrow$ P(6, 4, 6) and Q(2, 2, 2)

$$PQ:\ \frac{x-2}{2}=\frac{y-2}{1}=\frac{z-2}{2}=\lambda$$

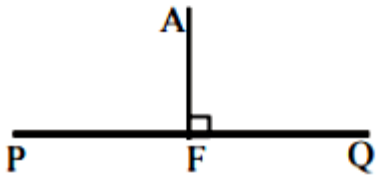
Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

A(1, 2, 12)

$\overrightarrow{AF}\cdot\overrightarrow{PQ}=0$

$\therefore\lambda=2$

So F(6, 4, 6) and AF = $\sqrt{65}$



Question44

Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of △PQR. Then, the angle ∠QPR is
[29-Jan-2024 Shift 2]

Options:

A. $\frac{\pi}{6}$

B. $\cos^{-1}\left(\frac{7}{18}\right)$

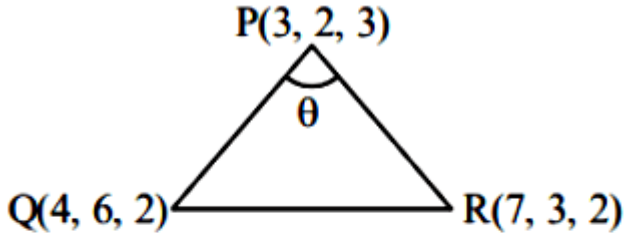
C. $\cos^{-1}\left(\frac{1}{18}\right)$

D. $\frac{\pi}{3}$

Answer: D

Solution:

Solution:



Direction ratio of PR = (4, 1, −1)

Direction ratio of PQ = (1, 4, −1)

Now, $\cos\theta = \left|\frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \frac{\pi}{3}$

Question45

Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\vec{OM}\cdot\vec{ON}$ is equal to [29-Jan-2024 Shift 2]

Answer: 9

Solution:

Solution:

$$L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda \quad \text{drs}(4, 1, 3) = b_1$$

$$M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$$

$$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$$

$$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$$

$$MN = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16) \ldots\ldots(1)$$

Now

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \ldots\ldots\ldots\ldots(2)$$

Equation (1) and (2)

$$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$$

I and II

$$\lambda - 5\mu + 6 = 0 \ldots\ldots\ldots(3)$$

I and III

$$\lambda - 3\mu + 4 = 0 \ldots\ldots\ldots\ldots(4)$$

Solve (3) and (4) we get

$$\lambda = -1, \mu = 1$$

$$\therefore M(1, 3, 2)$$

$$N(4, 3, -2)$$

$$\therefore \vec{OM} \cdot \vec{ON} = 4 + 9 - 4 = 9$$

Question46

Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. then 19(α + β + γ) is equal to :
[30-Jan-2024 Shift 1]

Options:

A. 102

B. 101

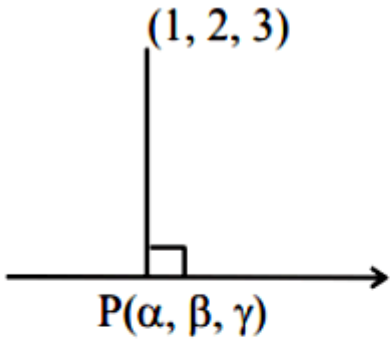
C. 99

D. 100

Answer: B

Solution:

Solution:



Let foot $P(5k-3, 2k+1, 3k-4)$

DR's $\longrightarrow$ AP: $5k-4, 2k-1, 3k-7$

DR's $\longrightarrow$ Line: 5, 2, 3

Condition of perpendicular lines $(25k-20)+(4k-2)+(9k-21)=0$

Then $k=\frac{43}{38}$

Then $19(\alpha+\beta+\gamma)=101$

Question47

If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value

of $\frac{32\sqrt{3}d_1}{d_2}$ is :__

[30-Jan-2024 Shift 1]

Answer: 16

Solution:

Solution:

$$L_1: \frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}, L_2: \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

$d_1 =$ shortest distance between L_1 & L_2

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|(\vec{b}_1 \times \vec{b}_2)|} \right|$$

$d_1 = 2$

$$L_3: \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, L_4: \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

$d_2 =$ shortest distance between L_3&L_4

$d_2 = \frac{12}{\sqrt{3}}$ Hence

$$= \frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \times 2}{\frac{12}{\sqrt{3}}} = 16$$

Question48

Let $L_1 : \vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 2\hat{k}\right), \lambda \in R$

$L_2 : \vec{r} = \left(\hat{j} - \hat{k}\right) + \mu\left(3\hat{i} + \hat{j} + p\hat{k}\right), \mu \in R$ and

$L_3 : \vec{r} = \delta\left(\ell\hat{i} + m\,\text{hat}\,j + n\hat{k}\right)\delta \in R$

Be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then the point which lies on L_3 is

[30-Jan-2024 Shift 2]

Options:

A. $(-1, 7, 4)$

B. $(-1, -7, 4)$

C. $(1, 7, -4)$

D. $(1, -7, 4)$

Answer: A

Solution:

Solution:

$L_1 \perp L_2$

$L_3 \perp L_1, L_2$

$3 - 1 + 2P = 0$

$P = -1$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore (-\delta, 7\delta, 4\delta)$ will lie on L_3

For $\delta = 1$ the point will be $(-1, 7, 4)$

Question49

Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at

N(a, b, c). Then the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals

[30-Jan-2024 Shift 2]

Answer: 196

Solution:

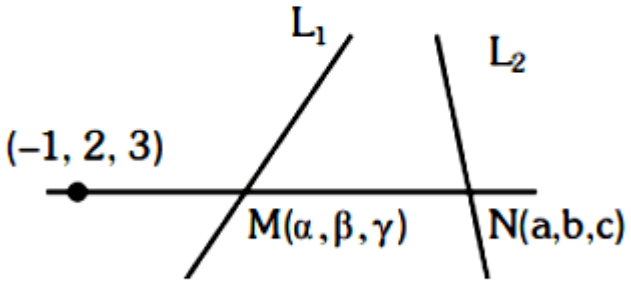
Solution:

$M(3\lambda+1, 2\lambda+2, -2\lambda-1)$

$\therefore \alpha+\beta+\gamma = 3\lambda+2$

$N(-3\mu-2, -2\mu+2, 4\mu+1)$

$\therefore a+b+c = -\mu+1$



$$\frac{3\lambda+2}{-3\mu-1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda-4}{4\mu-2}$$

$3\lambda\mu+2\mu = 3\lambda\mu+\lambda$

$2\mu = \lambda$

$2\lambda\mu-\lambda = \lambda\mu+2\mu$

$\lambda\mu = \lambda+2\mu$

$\Rightarrow \lambda\mu = 2\lambda$

$\Rightarrow \mu = 2 \quad (\lambda \neq 0)$

$\therefore \lambda = 4$

$\alpha+\beta+\gamma = 14$

$a+b+c = -1$

$$\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = 196$$

Question50

The distance of the point Q(0, 2, −2) form the line passing through the point P(5, −4, 3) and perpendicular to the lines $\vec{r} = \left(-3\hat{i} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$, $\lambda \in \mathbb{R}$ and $\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \mu\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$, $\mu \in \mathbb{R}$ is
[31-Jan-2024 Shift 1]

Options:

A. $\sqrt{86}$

B. $\sqrt{20}$

C. $\sqrt{54}$

D. $\sqrt{74}$

Answer: D

Solution:

Solution:
A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

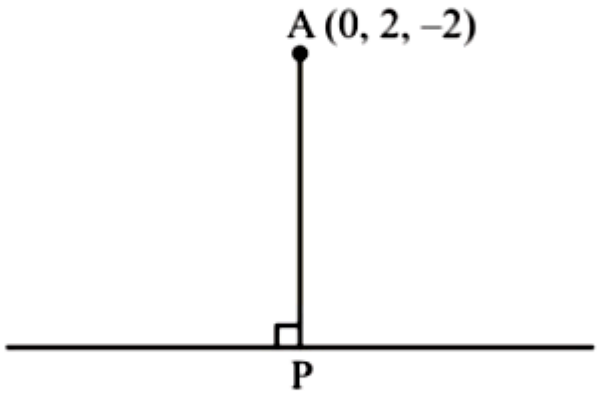
$$= -9\hat{i} - 9\hat{j} + 9\hat{k}$$

Required line,

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$$

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

Now distance of $(0, 2, -2)$



$$\text{P.V. of P} \equiv (5+\lambda)\hat{i} + (\lambda - 4)\hat{j} + (3-\lambda)\hat{k}$$

$$\vec{AP} = (5+\lambda)\hat{i} + (\lambda - 6)\hat{j} + (5-\lambda)\hat{k}$$

$$\vec{AP} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$$

$$\lambda = 2$$

$$|\vec{AP}| = \sqrt{49 + 16 + 9}$$

$$|\vec{AP}| = \sqrt{74}$$

Question51

Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = −y, z = −1 respectively. If ∠QPR is a

right angle, then $12a^2$ is equal to____
[31-Jan-2024 Shift 1]

Answer: 12

Solution:

Solution:

$$\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}=r\longrightarrow Q(r,r,1)$$

$$\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}=k\longrightarrow R(k,-k,-1)$$

$$\overline{PQ}=(a-r)\hat{i}+(a-r)\hat{j}+(a-1)\hat{k}$$

$$a=r+a-r=0$$

$$2a=2r\longrightarrow a=r$$

$$\overline{PR}=(a-k)i+(a+k)\hat{j}+(a+1)\hat{k}$$

$$a-k-a-k=0\Rightarrow k=0$$

As, PQ $\perp PR$

$$(a-r)(a-k)+(a-r)(a+k)+(a-1)(a+1)=0$$

$$a=1 \text{ or } -1$$

$$12a^2=12$$

Question52

Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.
Then $2\alpha+3\beta+4\gamma$ is equal to
[31-Jan-2024 Shift 2]

Options:

A. 32

B. 33

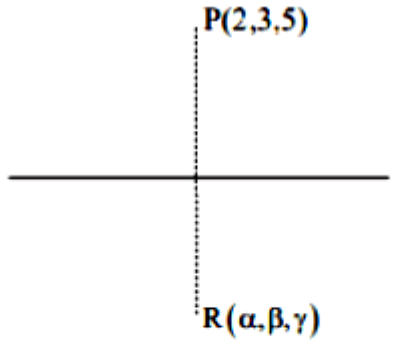
C. 31

D. 34

Answer: B

Solution:

Solution:



$\because \overrightarrow{PR} \perp (2, 3, 4)$

$\therefore \overrightarrow{PR} \cdot (2, 3, 4) = 0$

$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$

Question53

The shortest distance between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points A(−4, 4, 3) · B(−1, 6, 3) and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is
[31-Jan-2024 Shift 2]

Options:

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Answer: C

Solution:

Solution:

$$L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\therefore S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n_1} \times \vec{n_2}|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n_1} \times \vec{n_2}|}$$

$$= \frac{141}{\left|-4\hat{i}+6\hat{j}+13\hat{k}\right|}$$

$$= \frac{141}{\sqrt{16+36+169}}$$

$$= \frac{141}{\sqrt{221}}$$

Question54

A line passes through A(4, −6, −2) and B(16, −2, 4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the

points P(a, b, c) and Q(4, −12, 3) is equal to
[31-Jan-2024 Shift 2]

Answer: 22

Solution:

Solution:

$$\frac{x-4}{12}=\frac{x+6}{4}=\frac{z+2}{6}$$

$$\frac{x-4}{\frac{6}{7}}=\frac{y+6}{\frac{2}{7}}=\frac{z+2}{\frac{3}{7}}=21$$

$$\left(21\times\frac{6}{7}+4,\ \frac{2}{7}\times 21-6,\ \frac{3}{7}\times 21-2\right)$$

$$=(22,\ 0,\ 7)=(a,\ b,\ c)$$

$$\therefore \sqrt{324+144+16}=22$$

Question55

If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1 , then the sum of all possible values of λ is :____
[1-Feb-2024 Shift 1]

Options:

A. 0

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. $-2\sqrt{3}$

Answer: B

Solution:

Solution:

Passing points of lines $L_1 \& L_2$ are

$(\lambda, 2, 1) \& (\sqrt{3}, 1, 2)$

$$\text{S.D} = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left| \frac{\sqrt{3}-\lambda}{\sqrt{3}} \right|$$

$$\lambda = 0, \lambda = 2\sqrt{3}$$

Question56

Let the line of the shortest distance between the lines

$L_1 : \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

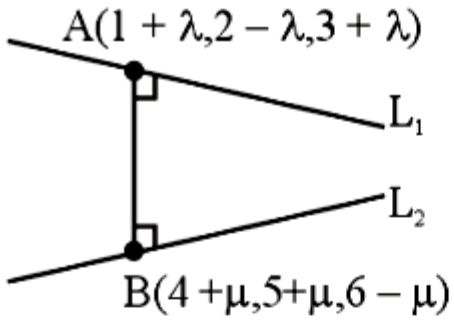
$L_2 : \vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to____
[1-Feb-2024 Shift 1]

Answer: 21

Solution:

Solution:



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$(DR's of L_1)

$\vec{d} = \hat{i} + \hat{j} - \hat{k}$(DR's of L_2)

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 0\hat{i} + 2\hat{j} + 2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0, 2, 2) = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda)$

$$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $A = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$B = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of $AB = \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

Question57

Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is:
[1-Feb-2024 Shift 2]

Options:

A. 26

B. 36

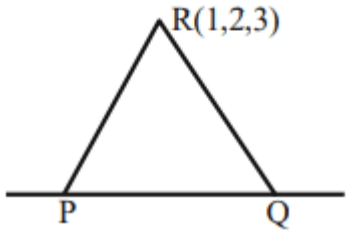
C. 18

D. 24

Answer: C

Solution:

Solution:



$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

$PR = 6$

$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$\lambda = 0, 1$

Hence $P(-3, 4, -1)$ & $Q(5, 6, 1)$

Centroid of $\Delta PQR = (1, 4, 1) \equiv (\alpha, \beta, \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = 18$

Question58

If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ), then $14(\alpha + \beta + \gamma)$ is : [1-Feb-2024 Shift 2]

Options:

A. 102

B. 138

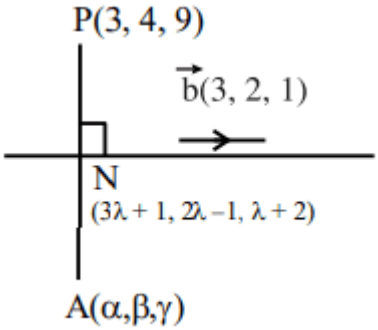
C. 108

D. 132

Answer: C

Solution:

Solution:



$\overrightarrow{PN} \cdot \overrightarrow{b} = 0?$

$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$

$N\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$

$\therefore \frac{\alpha + 3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$

$\frac{\beta + 4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$

$\frac{\gamma + 9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$

Ans. $14(\alpha + \beta + r) = 108$

Question59

The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is
[24-Jan-2023 Shift 1]

Options:

A. $13\sqrt{2}$

B. 31

C. 26

D. $20\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of line
$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$
G.P on line $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$
point of intersection of line & plane
$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$
$\lambda = 2$
Point $(5, 1, 8)$
Distance $= \sqrt{36 + 64 + 576} = 26$

Question60

The distance of the point (7, −3, −4) from the plane passing through the points (2, −3, 1), (−1, 1, −2) and (3, −4, 2) is : [24-Jan-2023 Shift 1]

Options:

A. 4

B. 5

C. $5\sqrt{2}$

D. $4\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of Plane is

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$$

$x - z - 1 = 0$
Distance of $P(7, -3, -4)$ from Plane is

$$d = \left| \frac{7+4-1}{\sqrt{2}} \right| = 5\sqrt{2}$$

Question61

The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to
[24-Jan-2023 Shift 1]

Answer: 14

Solution:

Solution:

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}$$

$$= \frac{16+12+168}{\left|-4\hat{i}+6\hat{j}-12k\right|} = \frac{196}{14} = 14$$

Question62

If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to
[24-Jan-2023 Shift 2]

Options:

A. -1

B. 3

C. 1

D. 5

Answer: D

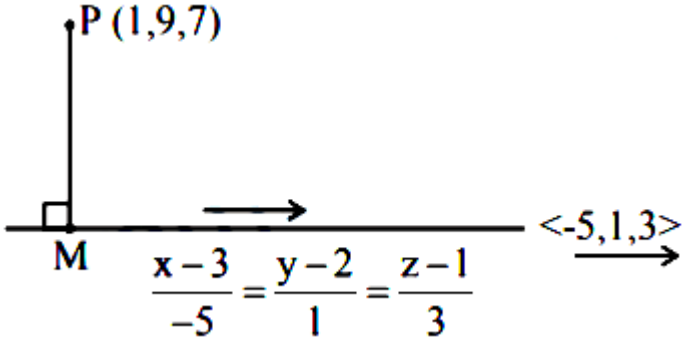
Solution:

Solution:

Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$



$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

Question63

Let the plane containing the line of intersection of the planes
P1 : $x + (\lambda + 4)y + z = 1$ and
P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1).
Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is
[24-Jan-2023 Shift 2]

**Let the plane containing the line of intersection of the planes
P1 : $x + (\lambda + 4)y + z = 1$ and
P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1).
Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is
[24-Jan-2023 Shift 2]**

Options:

A. $5\sqrt{6}$

B. $4\sqrt{6}$

C. $2\sqrt{6}$

D. $3\sqrt{6}$

Answer: D

Solution:

Solution:
Equation of plane passing through point of intersection of P1 and P2

$$P = P1 + kP2$$

$$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$$

Passing through $(0, 1, 0)$ and $(1, 0, 1)$

$$(\lambda + 4 - 1) + k(1 - 2) = 0$$

$$(\lambda + 3) - k = 0 \ldots (1)$$

Also passing $(1, 0, 1)$

$$(1 + 1 - 1) + k(2 + 1 - 2) = 0$$

$$1 + k = 0$$

$$k = -1$$

put in (1)

$$\lambda + 3 + 1 = 0$$

$$\lambda = -4$$

Then point $(2\lambda, \lambda, -\lambda)$

$$d = \left| \frac{-16 - 4, -4, 4)}{\sqrt{6}} \right|$$

$$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$$

Question64

If the shortest between the lines $\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}$ and $\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is
[24-Jan-2023 Shift 2]

Answer: 384

Solution:

Solution:
Shortest distance between the lines
$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}\ \ \frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{2+2\sqrt{6}}{5}$ is 6
Vector along line of shortest distance

$$=\begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}, \Rightarrow -\hat{i}+2\hat{j}-k$$ (its magnitude is $\sqrt{6}$)

Now $\frac{1}{\sqrt{6}}\begin{vmatrix} \sqrt{6}+\lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=\pm 6$

$\Rightarrow \lambda=-2\sqrt{6}, 10\sqrt{6}$
So, square of sum of these values is 384 .

Question65

Consider the lines L_1 and L_2 given by

$$L_1: \frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$$

$$L_2: \frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}$$

A line L_3 having direction ratios 1, −1, −2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is [25-Jan-2023 Shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{2}$

C. $4\sqrt{3}$

D. 4

Answer: A

Solution:

Solution:
Let $P = (2\lambda + 1, \lambda + 3, 2\lambda + 2)$
Let $Q = (\mu + 2, 2\mu + 2, 3\mu + 3)$

$\Rightarrow \frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$

$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8)$ and $Q(5, 8, 12)$

$PQ = 2\sqrt{6}$

Question66

The distance of the point P(4, 6, −2) from the line passing through the point (−3, 2, 3) and parallel to a line with direction ratios 3, 3, −1 is equal to :
[25-Jan-2023 Shift 1]

Options:

A. 3

B. $\sqrt{6}$

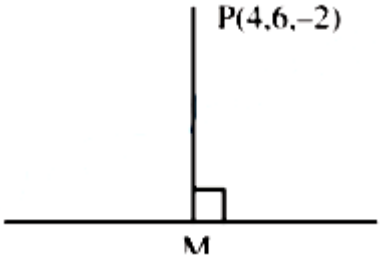
C. $2\sqrt{3}$

D. $\sqrt{14}$

Answer: D

Solution:

Solution:



Equation of line is $\frac{x+3}{3} = \frac{y-2}{3} = \frac{z-3}{-1} = \lambda$

$M(3\lambda - 3, 3\lambda + 2, 3 - \lambda)$

D.R of $PM(3\lambda - 7, 3\lambda - 4, 5 - \lambda)$

Since PM is perpendicular to line

$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$

$\Rightarrow \lambda = 2$

$\Rightarrow M(3, 8, 1) \Rightarrow PM = \sqrt{14}$

Question67

Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is _____.
[25-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:
Equation of plane is

$$(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow b = 12$$

$$\therefore \text{ plane is } 13x + 10y + 35z = 65$$

Distance from given point to plane $= 9$

Question68

The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then. Which of the following is NOT correct?
[25-Jan-2023 Shift 2]

Options:

A. $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

B. $\frac{\alpha}{\beta} = -8$

C. $\frac{\beta}{\gamma} = -5$

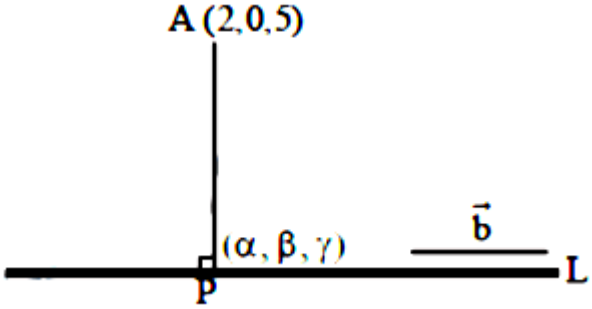
D. $\frac{\gamma}{\alpha} = \frac{5}{8}$

Answer: C

Solution:

Solution:

L : $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$ (let)



Let foot of perpendicular is

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$

Direction ratio of line $\Rightarrow \overrightarrow{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

Now, $\Rightarrow \overrightarrow{PA} \cdot \overrightarrow{b} = 0$

$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow \lambda = \frac{-1}{6}$

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \alpha = -\frac{4}{3}$

$\Rightarrow \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \beta = \frac{1}{6}$

$\Rightarrow \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \gamma = -\frac{5}{6}$

$\therefore$ Check options

Question69

The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is
[25-Jan-2023 Shift 2]

Options:

A. 2

B. 3

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

Solution:

$$\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}} \text{ and } \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

$$\Rightarrow \text{ Shortest distance } = \frac{\left(\vec{b}-\vec{a}\right)\cdot\left(\vec{p}\times\vec{q}\right)}{\left|\vec{p}\times\vec{q}\right|}$$

$$\text{S.D. } = \left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\frac{\left(\vec{p}\times\vec{q}\right)}{\left|\vec{p}\times\vec{q}\right|}$$

$$\left\{\vec{p}\times\vec{q} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{-1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \text{ or } 2\hat{i}-3\hat{j}+6\hat{k}\right\}$$

$$\text{S.D. } = \frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{2^2+3^2+6^2}} = \left|\frac{-14}{7}\right| = 2$$

Question70

If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _____.

[25-Jan-2023 Shift 2]

Answer: 18

Solution:

Solution:

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ $\vec{r} = \vec{a} + \lambda\vec{p}$

$\vec{r} = (+\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j})$ $\vec{r} = \vec{b} + \mu\vec{q}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$$d = \left| \frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}} \right|$$

$$= \left| \frac{-6+3}{\sqrt{14}} \right| = \frac{3}{\sqrt{14}}$$

$$\alpha = \frac{3}{\sqrt{14}}$$

Now, $28\alpha^2 = 2 \diagup 2 \times \frac{9}{14} = 18$

Question71

Let the co-ordinates of one vertex of $\triangle ABC$ be A(0, 2, α) and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of $\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____.
[29-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:

A. $(O_1 2, \alpha)$

$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$

$\Rightarrow 12\alpha^2+80\alpha+450 = 798$

$\Rightarrow 12\alpha^2+80\alpha-348 = 0$

$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$

Question72

Let the equation of the plane P containing the line $x+10 = \frac{8-y}{2} = z$ be $ax+by+3z = 2(a+b)$ and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2+b^2+c^2$ is equal to _____.
[29-Jan-2023 Shift 1]

Answer: 355

Solution:

Solution:

The line $\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$ have a point $(-10, 8, 0)$ with d. r. $(1, -2, 1)$

$\because$ the plane $ax+by+3z = 2(a+b)$

$\Rightarrow b = 2a$

& dot product of d.r.'s is zero

$\therefore a-2b+3 = 0$

$\therefore a = 1 \& b = 2$

Distance from $(1, 27, 7)$ is

$c = \frac{1+54+21-6}{\sqrt{14}} = \frac{70}{\sqrt{14}} = 5\sqrt{14}$

$\therefore a^2+b^2+c^2 = 1+4+350$

$= 355$

Question73

The plane $2x-y+z = 4$ intersects the line segment joining the points A(a, −2, 4) and B(2, b, −3) at the point C in the ratio 2 : 1 and the

distance of the point C from the origin is $\sqrt{5}$. If ab < 0 and P is the point (a − b, b, 2b − a) then CP^2 is equal to :
[29-Jan-2023 Shift 2]

Options:

A. $\frac{17}{3}$

B. $\frac{16}{3}$

C. $\frac{73}{3}$

D. $\frac{97}{3}$

Answer: A

Solution:

Solution:
A(a, −2, 4), B(2, b, −3)
AC : CB = 2 : 1
$\Rightarrow C \equiv \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3} \right)$
C lies on $2x - y + 2 = 4$
$\Rightarrow \frac{2a+8}{3} - \frac{2b-2}{3} - \frac{2}{3} = 4$
$\Rightarrow a - b = 2 \ldots$ (1)
Also $OC = \sqrt{5}$
$\Rightarrow \left(\frac{a+4}{3} \right)^2 + \left(\frac{2b-2}{3} \right)^2 + \frac{4}{9} = 5$ . . . (2)
Solving, (1) and (2)
$(b+6)^2 + (2b-2)^2 = 41$
$\Rightarrow 5b^2 + 4b - 1 = 0$
$\Rightarrow b = -1 \text{ or } \frac{1}{5}$
$\Rightarrow a = 1 \text{ or } \frac{11}{5}$
But $ab < 0 \Rightarrow (a, b) = (1, -1)$
$C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3} \right)$, $P \equiv (2, -1, -3)$
$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$

Question74

Shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is
[29-Jan-2023 Shift 2]

Options:

A. $2\sqrt{3}$

B. $4\sqrt{3}$

C. $3\sqrt{3}$

D. $5\sqrt{3}$

Answer: B

Solution:

Solution:

$\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}\ \vec{a}=i-8\hat{j}+4\hat{k}$

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}\ \vec{b}=\hat{i}+2\hat{j}+6\hat{k}$

$\vec{p}=2\hat{i}-7\hat{j}+5\hat{k},\ \vec{q}=2\hat{i}+\hat{j}-3\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}$$

$=\hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$

$=16\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=\left|\frac{-12}{\sqrt{3}}\right|=4\sqrt{3}|$

Question75

If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersects at the point P, then the distance of the point P from the plane z = a is :
[29-Jan-2023 Shift 2]

Options:

A. 16

B. 28

C. 10

D. 22

Answer: B

Solution:

Solution:
Point on $L_1 \equiv (\lambda + 1, 2\lambda + 2, \lambda - 3)$
Point on $L_2 \equiv (2\mu + a, 3\mu - 2, \mu + 3)$
$\lambda - 3 = \mu + 3 \Rightarrow \lambda = \mu + 6$ . . . (1)
$2\lambda + 2 = 3\mu - 2 \Rightarrow 2\lambda = 3\mu - 4$ . . . (2)
Solving, (1) and (2)
$\Rightarrow \lambda = 22 \& \mu = 16$
$\Rightarrow P \equiv (23, 46, 19)$
$\Rightarrow a = -9$
Distance of P from $z = -9$ is 28

Question76

Let a unit vector $\overset{\frown}{OP}$ make angle α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$ $\overset{\frown}{OP}$ is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true ?
[30-Jan-2023 Shift 1]

Options:

A. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

B. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

C. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

D. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Answer: A

Solution:

Solution:
Equation of plane :-

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow [x-1] - 4[y-2] + 3[z-3] = 0$
$\Rightarrow x - 4y + 3z = 2$
D.R's of normal of plane $<1, -4, 3>$
D.C's of $\left\langle \pm \frac{1}{\sqrt{26}}, \mp \frac{4}{\sqrt{26}}, \pm \frac{3}{\sqrt{26}} \right\rangle$

$\cos\beta = \frac{4}{\sqrt{26}}$

$\cos\alpha = \frac{-1}{\sqrt{26}} \quad \frac{\pi}{2} < \alpha < \pi$

$\cos\gamma = \frac{-3}{\sqrt{26}} \quad \frac{\pi}{2} < \gamma < \pi$

Question77

The line l_1 passes through the point (2, 6, 2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :
[30-Jan-2023 Shift 1]

Options:

A. 7

B. $\frac{19}{3}$

C. $\frac{19}{3}$

D. 9

Answer: D

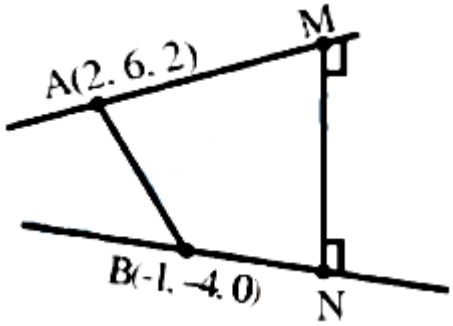
Solution:

Solution:
Line ℓ, is given by

$$L_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$$

Given,

$$L_2 : \frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$$



$$\text{Shortest distance} = \left| \overrightarrow{AB} \cdot \frac{\overrightarrow{MN}}{MN} \right|$$

$$\overline{AB} = 3\hat{i} + 10\hat{j} + 2\hat{k}$$

$$\overline{MN} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$$

$$MN = \sqrt{16 + 64 + 64} = 12$$

$$\therefore \quad \text{Shortest distance} = \left| \frac{-12 - 80 - 16}{12} \right| = 9$$

$\therefore$ Option (4) is correct.

Question78

If the equation of the plane passing through the point (1, 1, 2) and perpendicular to the line $x - 3y + 2z - 1 = 0$ $4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to _____.
[30-Jan-2023 Shift 1]

Answer: 15

Solution:

Solution:

$$x-3y+2z-1=0$$
$$4x-y+z=0$$

$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix}$$

$$= -\hat{i} + 7\hat{j} + 11\hat{k}$$

Dre of normal to the plane is $-1, 7, 11$

Equation of plane :

$$-1(x-1)+7(y-1)+11(z-2)=0$$
$$-x+7y+11z=28$$
$$\frac{-1}{28}x+\frac{7y}{28}+\frac{11z}{28}=1$$
$$Ax+By+Cz=1$$
$$140(C-B+A)=140\left(\frac{11}{28}-\frac{7}{28}-\frac{1}{28}\right)$$
$$=140\times\frac{3}{28}=15$$

Question79

If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \bar{r}\left(3\hat{i}-5\hat{j}+\hat{k}\right)=7$ and $P_2 : \vec{r}\cdot\left(\lambda\hat{i}+\hat{j}-3\hat{k}\right)=9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _____.

[30-Jan-2023 Shift 1]

Answer: 315

Solution:

Solution:

$$P_1 = \vec{r}\cdot\left(3\hat{i}-5\hat{j}+\hat{k}\right)=7$$
$$P_2 = \vec{r}\cdot\left(\lambda\hat{i}+\hat{j}-3\hat{k}\right)=9$$
$$\theta=\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$

$\therefore \cos\theta = \frac{1}{5}$

$\cos\theta = \vec{r} \cdot \frac{\vec{r}}{|\bar{r}_1||\vec{r}_2|}$

$= \frac{(3i-5j+K)(\lambda i+j-3K)}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}$

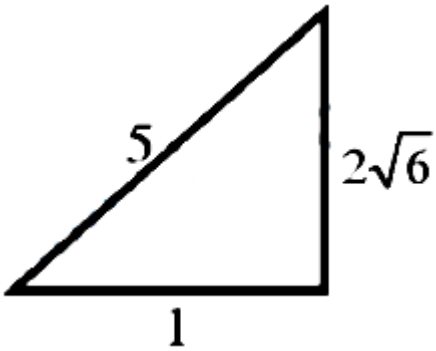
$\frac{1}{5} = \left|\frac{3\lambda-8}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}\right|$

Square $\Rightarrow \frac{1}{25} = \frac{9\lambda^2+64-48\lambda}{35(\lambda^2+10)}$

$\Rightarrow 19\lambda^2-120\lambda+125=0$

$\Rightarrow 19\lambda^2-95\lambda-25\lambda+125=0$

$\Rightarrow x=5,\ \frac{25}{19}$



Perpendicular distance of point
$(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$ from plane P_1

$= \frac{|3\times 50-5\times 50+2-7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$

Square $= \frac{105\times 105}{35} = 315$

Question80

A vector $\vec{v}$ in the first octant is inclined to the x axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c), is normal to $\vec{v}$, then [30-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}a+b+c=1$

B. $a+b+\sqrt{2}c=1$

C. $a+\sqrt{2}b+c=1$

D. $\sqrt{2}a-b+c=1$

Answer: C

Solution:

Solution:

$\hat{v}=\cos 60^\circ \hat{i}+\cos 45^\circ \hat{j}+\cos\gamma \hat{k}$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1 \quad (\gamma \rightarrow \text{ Acute })$

$\Rightarrow \cos\gamma=\frac{1}{2}$

$\Rightarrow \gamma=60^\circ$

Equation of plane is

$\frac{1}{2}(x-\sqrt{2})+\frac{1}{\sqrt{2}}(y+1)+\frac{1}{2}(z-1)=0$

$\Rightarrow x+\sqrt{2}y+z=1$

(a, b, c) lies on it.

$\Rightarrow a+\sqrt{2}b+c=1$

Question81

If a plane passes through the points (−1, k, 0), (2, k, −1), (1, 1, 2) and is parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is [30-Jan-2023 Shift 2]

Options:

A. $\frac{17}{5}$

B. $\frac{5}{17}$

C. $\frac{6}{13}$

D. $\frac{13}{6}$

Answer: D

Solution:

Solution:

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$$

$$\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$$

Points : A $(-1, k, 0)$, B$(2, k, -1)$, C$(1, 1, 2)$

$$\overrightarrow{CA} = -2\hat{i} + (k-1)\hat{j} - 2\hat{k}$$

$$\overrightarrow{CB} = \hat{i} + (k-1)\hat{j} - 3\hat{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & k-1 & -2 \\ 1 & k-1 & -3 \end{vmatrix}$$

$$= \hat{i}(-3k+3+2k-2) - \hat{j}(6+2) + \hat{k}(-2k+2-k+1)$$

$$= (1-k)\hat{i} - 8\hat{j} + (3-3k)\hat{k}$$

The line $\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$ is perpendicular to normal vector.

$$\therefore 1 \cdot (1-k) + 1(-8) + (-1)(3-3k) = 0$$

$$\Rightarrow 1 - k - 8 - 3 + 3k = 0$$

$$\Rightarrow 2k = 10 \Rightarrow k = 5$$

$$\therefore \frac{k^2+1}{(k-1)(k-2)} = \frac{26}{4 \cdot 3} = \frac{13}{6}$$

Question82

Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _____.
[30-Jan-2023 Shift 2]

Answer: 158

Solution:

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

$\therefore$ Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be $(5, 3, 8)$ and foot of $\perp$ from Q on this line be R.

Now, $R \equiv (k+2, -k+3, -k+1)$

DR of QR are $(k-3, -k, -k-7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

Question83

Let the shortest distance between the lines $L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}$, $\lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible? [31-Jan-2023 Shift 1]

Options:

A. $\alpha + 2\gamma = 24$

B. $2\alpha + \gamma = 7$

C. $2\alpha - \gamma = 9$

D. $\alpha - 2\gamma = 19$

Answer: A

Solution:

Solution:

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$$

$$a_2 - a_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$$

$2\sqrt{6} = \left| \frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}} \right|$

$|\lambda+3| = 12 \Rightarrow \lambda = 9, -15$

$\alpha = -2k+5, \gamma = k-\lambda$ where $k \in R$

$\Rightarrow \alpha+2\gamma = 5-2\lambda = -13, 35$

Question84

Let θ be the angle between the planes $P_1 = \vec{r} \cdot \left(\hat{i}+\hat{i}+2\hat{k}\right) = 9$ and $P_2 = \vec{r} \cdot \left(2\hat{i}-\hat{i}+\hat{k}\right) = 15$.

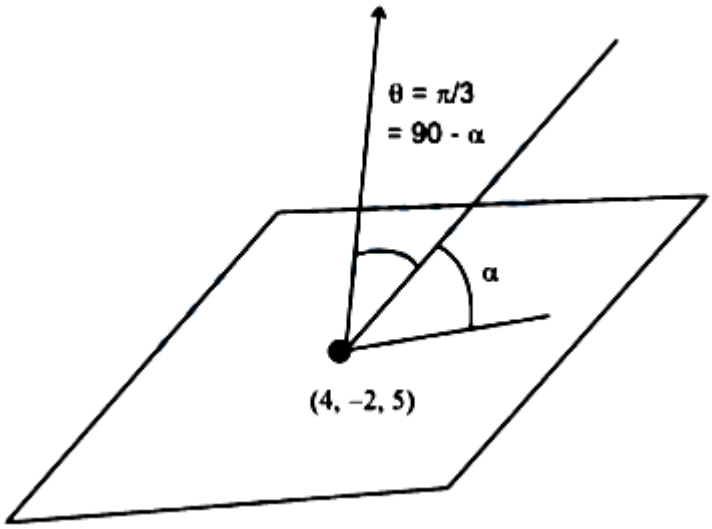
Let L be the line that meets P_2 at the point (4, −2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2 then $(\tan^2\theta)(\cot^2\alpha)$ is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:



$\cos\theta = \frac{\left(\hat{i}+\hat{j}+2\hat{k}\right)\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)}{6} = \frac{2-1+2}{6} = \frac{1}{2}$

$\theta = \pi/3 \quad \alpha = \pi/6$

$(\tan^2\theta)(\cot^2\alpha)$
(3) (3)=9

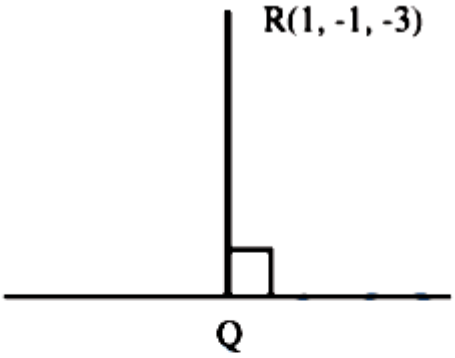
Question85

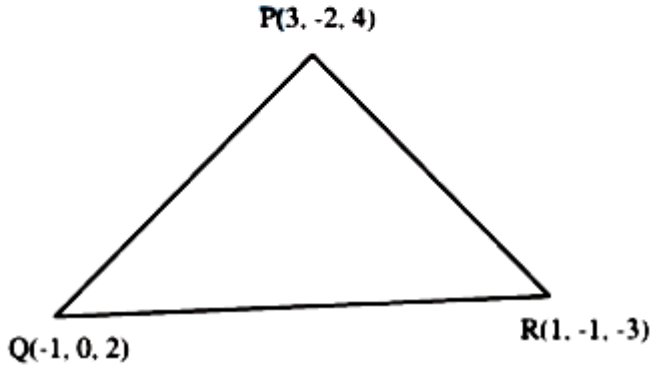
Let the line L : $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point R(1, −1, −3) on the line L. If α is the area of triangle PQR. then α^2 is equal to _____.
[31-Jan-2023 Shift 1]

Answer: 180

Solution:

Solution:
Any point on $L((2\lambda + 1), (-\lambda - 1), (\lambda + 3))$
$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$
$6\lambda + 10 = 16 \Rightarrow \lambda = 1$
$\therefore P = (3, -2, 4)$
DR of QR = $\langle 2\lambda, -\lambda, \lambda + 6\rangle$
DR of L = $\langle 2, -1, 1\rangle$
$4\lambda + \lambda + \lambda + 6 = 0 \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$
$Q = (-1, 0, 2)$





$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$ $\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$

Question86

If a point P(α, β, γ) satisfying $(\alpha\beta\gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0\ 0\ 0)$ lies on the plane 2x + 4y + 3z = 5, then 6α + 9β + 7γ is equal to:
[31-Jan-2023 Shift 2]

Options:

A. -1

B. $\frac{11}{5}$

C. $\frac{5}{4}$

D. 11

Answer: D

Solution:

Solution:
$2\alpha + 4\beta + 3\gamma = 5$ . . . (1)
$2\alpha + 9\beta + 8\gamma = 0$ . . . (2)
$10\alpha + 3\beta + 4\gamma = 0$ . . . (3)
$8\alpha + 8\beta + 8\gamma = 0$ . . . (4)
Subtract (4) from (2)
$-6\alpha + \beta = 0$
$\beta = 6\alpha$ . . . (5)
From equation (4)
$8\alpha + 48\alpha + 8\gamma = 0$
$\gamma = -7\alpha$ . . . (6)
From equation (1)
$2\alpha + 24\alpha - 21\alpha = 5$
$5\alpha = 5$
$\alpha = 1$
$\beta = +6, \quad \gamma = -7$
$\therefore 6\alpha + 9\beta + 7\gamma$
$= 6 + 54 - 49$
$= 11$

Question87

Let the plane P : $8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line L : $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :
[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{14}$

B. $\frac{6}{\sqrt{14}}$

C. $\sqrt{\frac{2}{7}}$

D. $\sqrt{\frac{7}{2}}$

Answer: A

Solution:

Solution:
P: $8x + \alpha_1 y + \alpha_2 z + 12 = 0$
L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$
$\because$P is parallel to L
$\Rightarrow 8(2) + \alpha_1(3) + 5(\alpha_2) = 0$
$\Rightarrow 3\alpha_1 + 5(\alpha_2) = -16$
Also y-intercept of plane P is 1
$\Rightarrow \alpha_1 = -12$
And $\alpha_2 = 4$
$\Rightarrow$ Equation of plane P is $2x - 3y + z + 3 = 0$
$\Rightarrow$ Distance of line L from Plane P is
$= \left| \frac{0 - 3(6) + 1 + 3}{\sqrt{4+9+1}} \right|$
$= \sqrt{14}$

Question88

Let P be the plane, passing through the point (1, −1, −5) and perpendicular to the line joining the points (4, 1, −3) and (2, 4, 3). Then the distance of P from the point (3, −2, 2) is [31-Jan-2023 Shift 2]

Options:

A. 6

B. 4

C. 5

D. 7

Answer: C

Solution:

Solution:
Equation of Plane :
$2(x-1) - 3(y+1) - 6(z+5) = 0$
Or $2x - 3y - 6z = 35$

$\Rightarrow$ Re quired distance $= \frac{|2(3)-3(-2)-6(2)-35|}{\sqrt{4+9+36}}$

$= 5$

Question89

The shortest distance between the lines $\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and $\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is [1-Feb-2023 Shift 1]

Options:

A. $7\sqrt{3}$

B. $5\sqrt{3}$

C. $6\sqrt{3}$

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:
Shortest distance between two lines
$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3\&}$
$\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3}$ is given as

$$\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{(a_1b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$\frac{\begin{vmatrix} 5-(3) & 2-(-5) & 4-1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(-10+12)^2+(-5+3)^2+(4-2)^2}}$$

$$\frac{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}}$$

$$= \frac{|8(-10+12)-7(-5+3)+3(4-2)|}{\sqrt{4+4+4}}$$

$$= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

Question90

Let the image of the point P(2, −1, 3) in the plane x + 2y − z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is
[1-Feb-2023 Shift 1]

Options:

A. $\frac{22\sqrt{2}}{7}$

B. $\frac{24\sqrt{2}}{7}$

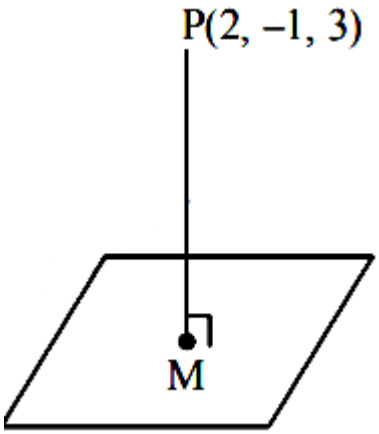
C. $2\sqrt{14}$

D. $3\sqrt{14}$

Answer: D

Solution:

Solution:



eq. of line PM $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\lambda$

any point on line $=(\lambda+2, 2\lambda-1, -\lambda+3)$

for point ' m ' $(\lambda+2)+2(2\lambda-1)-(3-\lambda)=0$

$\lambda=\frac{1}{2}$

Point $m\left(\frac{1}{2}+2, 2\times\frac{1}{2}-1, \frac{-1}{2}+3\right)$

$=\left(\frac{5}{2}, 0, \frac{5}{2}\right)$

For Image $Q(\alpha, \beta, \gamma)$

$\frac{\alpha+2}{2}=\frac{5}{2}, \frac{\beta-1}{2}=0,$

$\frac{\gamma+3}{2}=\frac{5}{2}$

$Q : (3, 1, 2)$

$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$

$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$

Question91

Let the plane P pass through the intersection of the planes $2x+3y-z=2$ and $x+2y+3z=6$, and be perpendicular to the plane $2x+y-z+1=0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to :

[1-Feb-2023 Shift 2]

Options:

A. $\frac{250}{83}$

B. $\frac{15}{53}$

C. $\frac{25}{83}$

D. $\frac{250}{82}$

Answer: A

Solution:

Solution:

$P \equiv P_1 + \lambda P_2 = 0$

$(2+\lambda)x + (3+2\lambda)y + (3\lambda-1)z - 2 - 6\lambda = 0$

Plane P is perpendicular to $P_3 \therefore \vec{n} \cdot \vec{n}_3 = 0$

$2(\lambda+2) + (2\lambda+3) - (3\lambda-1) = 0$

$\lambda = -8$

$P \equiv -6x - 13y - 25z + 46 = 0$

$6x + 13y + 25z - 46 = 0$

Dist from $(-7, 1, 1)$

$d = \left| \frac{-42 + 13 + 25 - 46}{\sqrt{36 + 169 + 625}} \right| = \frac{50}{\sqrt{830}}$

$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$

Question92

The point of intersection C of the plane 8x + y + 2z = 0 and the line joining the points A(−3, −6, 1) and B(2, 4, −3) divides the line segment AB internally in the ratio k : 1. If a, b, c (|a|, | b | , | c| are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then |a + b + c| is equal to _____. [1-Feb-2023 Shift 2]

Answer: 10

Solution:

Solution:

Plane: $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

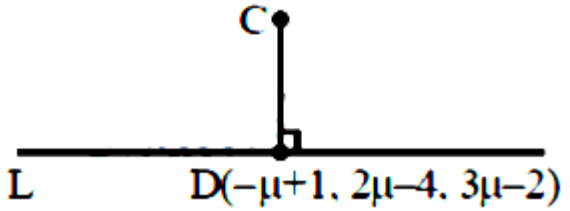
Point of intersection of line and plane

$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

L : $\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overrightarrow{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

Direction ratios $\rightarrow (-1, -26, 17)$

$|a + b + c| = 10$

Question93

Let αx + βy + yz = 1 be the equation of a plane passing through the point (3, −2, 5) and perpendicular to the line joining the points (1, 2, 3) and (−2, 3, 5). Then the value of αβy is equal to _____. [1-Feb-2023 Shift 2]

Answer: 6

Solution:

Solution:

Given Equation is not equation of plane as yz is present. If we consider y is γ then answer would be 6 .

Normal vector of plane $= 3\hat{i} - \hat{j} - 2\hat{k}$
Plane : $3x - y - 2z + \lambda = 0$
Point $(3, -2, 5)$ satisfies the plane
$\lambda = -1$
$3x - y - 2z = 1$
$\alpha\beta y = 6$

Question94

One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3,4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :
[6-Apr-2023 shift 1]

Options:

A. $\frac{12}{5\sqrt{5}}$

B. $12\sqrt{5}$

C. $\frac{12}{5}$

D. $\frac{12}{\sqrt{5}}$

Answer: C

Solution:

Solution:
Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$
$a_1 = (0, 0, 0)$ $a_2 = (3, 0, 5)$
$b_1 = (3, 4, 5)$ $b_2 = (0, 0, 1)$
Equation of edge parallel to z axis
$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$

$$S.D = \frac{\left(\vec{a}_2 \cdot \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$\frac{\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{3(4)}{\left|4\hat{i} - 3\hat{j}\right|} = \frac{12}{5}$$

Question95

If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to :

[6-Apr-2023 shift 1]

Options:

A. 15

B. 14

C. 13

D. 12

Answer: B

Solution:

Solution:
Using family of planer
$P : P_1 + \lambda P_2 = 0 \Rightarrow P(2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z = (3 - 9\lambda)$
P is | to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$
Then for $\lambda : \vec{n}_p \cdot \vec{v}_L = 0$
$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$
$-3 + 5\lambda = 0 \Rightarrow \lambda = \frac{3}{5}$
Hence : $P : 22x - 14y + 20z = -12$

$P: 11x - 7y + 10z + 6 = 0$
$\Rightarrow a = 11$
$b = -7$
$c = 10$
$\Rightarrow a + b + c = 14$

Question96

Let the image of the point P(1, 2, 3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7). then the square of the area of the triangle PQR is _____.
[6-Apr-2023 shift 1]

Answer: 594

Solution:

Solution:
Let $Q(\alpha, \beta, \gamma)$ be the image of P, about the plane
$2x - y + z = 9$
$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$
$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$
Then area of triangle PQR is $= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$
$= \left|-12\hat{i} - 3\hat{j} + 21\hat{k}\right| = \sqrt{144 + 9 + 441} = \sqrt{594}$
Square of area $= 594$

Question97

Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point P(1, −9, 2) from the line L is :
[6-Apr-2023 shift 2]

Options:

A. 9

B. $\sqrt{54}$

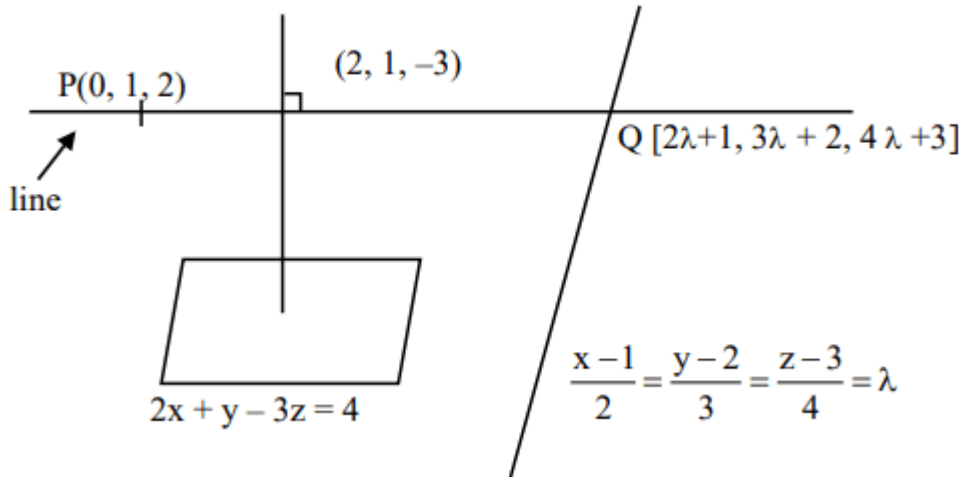
C. $\sqrt{69}$

D. $\sqrt{74}$

Answer: D

Solution:

Solution:



$\overrightarrow{PQ} = (2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$

$\overrightarrow{PQ} \cdot \overrightarrow{n} = 0 \quad \Rightarrow (2\lambda + 1) \cdot (2) + (3\lambda + 1)(1) + (4\lambda + 1)(-3) = 0$

$\Rightarrow -5\lambda = 0$

$\Rightarrow \lambda = 0$

$Q = (1, 2, 3)$

$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \mu$

distance of line from $(1, -9, 2)$

$(P'Q) \cdot (1, 1, 1) = 0$

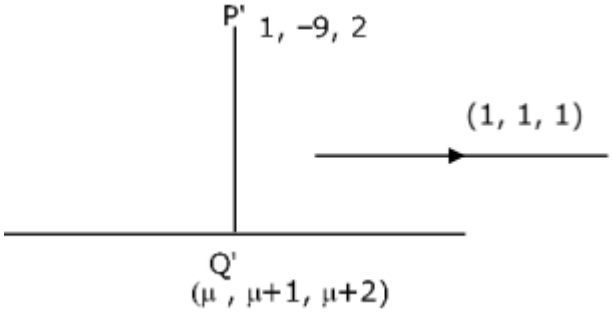
$\Rightarrow [\mu - 1, \mu + 10, \mu] \cdot [1, 1, 1] = 0$

$\Rightarrow \mu - 1 + \mu + 10 + \mu = 0$

$\mu = -3$

$Q' = (-3, -2, 1)$

$P'Q' = \sqrt{16 + 49 + 9} = \sqrt{74}$



Question98

A plane P contains the line of intersection of the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 6$ and $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = -5$. If P passes through the point (0, 2, −2), then the square of distance of the point (12, 12, 18) from the plane P is :
[6-Apr-2023 shift 2]

Options:

A. 620

B. 1240

C. 310

D. 155

Answer: A

Solution:

Solution:
eq ^{n} of plane $P_1 + \lambda P_2 = 0$
$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$
pass th. $(0, 2, -2)$
$(-6) + \lambda(6 - 8 + 5) = 0$
$(-6) + \lambda[3] = 0 \Rightarrow \lambda = 2$
eq^{n} of plane
$5x + 7y + 9z + 4 = 0$
distance from $(12, 12, 18)$
$d = \left| \frac{60 + 84 + 162 + 4}{\sqrt{25 + 49 + 81}} \right|$
$d = \frac{310}{\sqrt{155}}$
$d^2 = \frac{310 \times 310}{155}$
$d^2 = 620$
Ans. Option 1

Question99

If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____.
[6-Apr-2023 shift 2]

Answer: 18

Solution:

Solution:
If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect Point of first line $(1, 2, 3)$ and point on second line $(4, 1, 0)$.
Vector joining both points is $-3\hat{i}+\hat{j}+3k$
Now vector along second line is $2\hat{i}+3\hat{j}+\alpha k$
Also vector along second line is $5\hat{i}+2\hat{j}+\beta k$
Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix}$$

$\Rightarrow 2(6-\beta)-3(15+3\beta)+\alpha(11)=0$
$\Rightarrow \alpha-\beta=3$
Now $\alpha=3+\beta$
Given expression $8(3+\beta)\cdot\beta=8(\beta^2+3\beta)$
$=8\left(\beta^2+3\beta+\frac{9}{4}-\frac{9}{4}\right)=8\left(\beta+\frac{3}{2}\right)^2-18$
So magnitude of minimum value $=18$

Question100

The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is
[8-Apr-2023 shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{6}$

C. $6\sqrt{3}$

D. $6\sqrt{2}$

Answer: B

Solution:

Solution:

$$S_d = \left| \frac{\left(\vec{a} - \vec{b}\right) \times \left(\vec{n}_1 \times \vec{n}_2\right)}{\left|\vec{n}_1 \times \vec{n}_2\right|} \right|$$

$\bar{a} = (4, -2, -3)$
$\bar{b} = (1, 3, 4)$
$\bar{n}_1 = (4, 5, 3)$
$\bar{n}_2 = (3, 4, 2)$

$$\bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2, 1, 1)$$

$$S_d = \frac{(3, -5, -7) \cdot (-2, 1, 1)}{\sqrt{6}} = \left| \frac{-6 - 5 - 7}{\sqrt{6}} \right| = 3\sqrt{6}$$

Question101

If the eqation of the plane containing the line $x + 2y + 3z - 4 = 02x + y - z + 5$ and perpendicular to the plane $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ is ax+by +cz = 4, then $(a - b + c)$ is equal to
[8-Apr-2023 shift 1]

Options:

A. 22

B. 24

C. 20

D. 18

Answer: A

Solution:

Solution:

D.R's of line $\vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$

D.R's of normal of second plane

$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$

A point on the required plane is $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$

The equation of required plane is

$27x + 30y + 25z = 4$

$\therefore a - b + c = 22$

Question102

Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is _____.

[8-Apr-2023 shift 1]

Answer: 9

Solution:

Solution:

$2x + 3y - 6z + 7 = 0 \left(\frac{5}{2}, 1, \lambda\right), (-2, 0, 1)$

$d_1 = \left|\frac{5 + 3 - 6\lambda + 7}{7}\right| = d_2 = \left|\frac{-4 - 6 + 7}{7}\right|$

$\Rightarrow |15 - 6\lambda| = |3|$

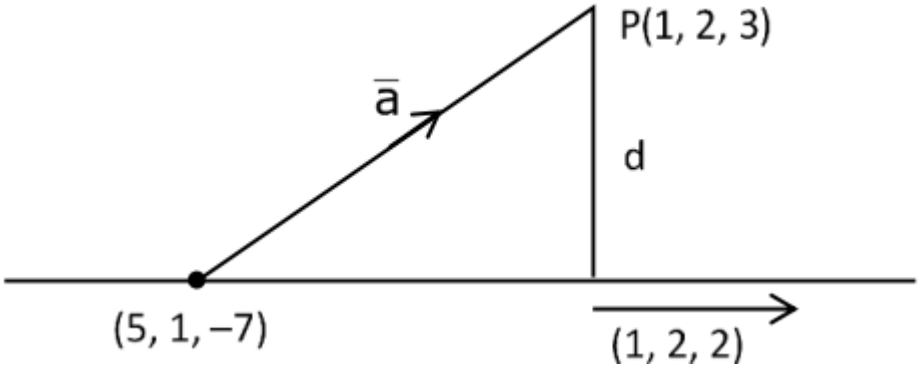
$15 - 6\lambda = 3 \text{ or } 15 - 6\lambda = -3$

$6\lambda = 12 \quad 6\lambda = 18$

$\lambda = 2 \quad \lambda = 3$

$\lambda_1 = 3, \quad \lambda_2 = 2$

$\therefore P(1, 2, 3) \quad \frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$



$$d = \left| \frac{(4,-1,-10)\times(1,2,2)}{3} \right| = \left| \frac{18\hat{i}-18\hat{j}+9\hat{k}}{3} \right| = 9$$

Question103

For a, b ∈ Z and |a − b| ≤ 10, let the angle between the plane P : ax + y − z = b and the line 1 : x − 1 = a − y = z +1 be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, −6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 85

B. 48

C. 25

D. 32

Answer: D

Solution:

Solution:

$$\theta = \cos^{-1}\frac{1}{3} \therefore \sin\theta = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin\theta = \frac{a\cdot 1 + 1(-1) + (-1)\cdot 1}{\sqrt{a^2+1+1}\cdot\sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \{3(a-2)\}^2 = 24(a^2+2)$$

$$\Rightarrow 3(a^2-4a+4) = 8a^2+16$$

$\Rightarrow 5a^2 + 12a + 4 = 0$

$\Rightarrow 5a^2 + 10a + 2a + 4 = 0$

$\therefore a = -2, \ \frac{-2}{5} \because a \in z$

$\therefore a = -2$

Distance of $(6, -6, 4)$ from $-2x + y - z - b = 0$ is $3\sqrt{6}$

$\therefore \left| \frac{-12 - 6 - 4 - b}{\sqrt{4 + 1 + 1}} \right| = 3\sqrt{6}$

$\Rightarrow |b + 22| = 18 \therefore b = -40, -4$

$\because |a - b| \leq 10$

$\therefore b = -4$

$\therefore a^4 + b^2$

$= 32$ Ans.

Question104

Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

[8-Apr-2023 shift 2]

Answer: 11

Solution:

Solution:

P_2 is given by

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$x - y - z = 3$

DR of line intersection of $P_1 \& P_2$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$+6\hat{i}+4\hat{j}+2\hat{k}$

Let

$z=0, \quad x-y=3$

$3x-y=112x=8$

$x=4$

$y=1$

So Line is

$$\frac{x-4}{6}=\frac{y-1}{4}=\frac{z-0}{2}=r$$

$(\alpha, \beta, \gamma)=(6r+4, 4r+1, 2r)$

$6(\alpha-7)+4(\beta-4)+2(\gamma+1)=0$

$6\alpha-42+4\beta-16+2\gamma+2=0$

$36r+24+16r+4+4r-56=0$

$56r=28$

$r=\frac{1}{2}$ $\alpha+\beta+\gamma=12r+5$

$=6+5=11$

Question105

Let P be the plane passing through the line $\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z+5}{7}$ and the point (2, 4, −3). If the image of the point (−1, 3, 4) in the plane P is (α, β, γ) then α + β + γ is equal to
[8-Apr-2023 shift 2]

Options:

A. 12

B. 9

C. 10

D. 11

Answer: C

Solution:

Solution:

Equation of plane is given by

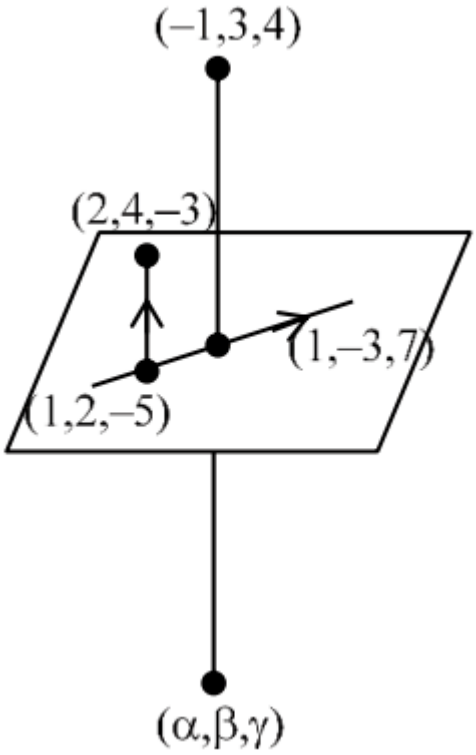
$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$

$4x - y - z = 7$

$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$

$\alpha = 7, \beta = 1, \gamma = 2$

$\alpha + \beta + \gamma = 10$(Option 3)



Question106

Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}$, $2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :

[10-Apr-2023 shift 1]

Options:

A. $\frac{10}{3}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. 3

Answer: D

Solution:

Solution:
Position vector of the point P(−1, −2, 3), A(−2, 1, −3)B(2, 4, −2), and C(−4, 2, −1)

Then $\overrightarrow{OP}\cdot\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC})|}$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 3 & 1\\ -2 & 1 & 2\end{vmatrix}$$

$$=\hat{i}(5)-\hat{j}(8+2)+\hat{k}(4+6)$$
$$=5\hat{i}-10\hat{j}+10\hat{k}$$

Now

$$\overrightarrow{OP}\cdot\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC})|}=\left(-\hat{i}-2\hat{j}+3\hat{k}\right)\cdot\frac{\left(\hat{i}-10\hat{j}+10\hat{k}\right)}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$
$$=\frac{-5+20+30}{\sqrt{25+100+100}}$$
$$=\frac{45}{\sqrt{225}}=\frac{45}{15}=3$$

Question107

Let two vertices of a triangle ABC be (2, 4, 6) and (0, −2, −5), and its centroid be (2, 1, −1). If the image of the third vertex in the plane x + 2y + 4z = 11 is (α, β, γ), then αβ + βγ + γα is equal to : [10-Apr-2023 shift 1]

Options:

A. 76

B. 74

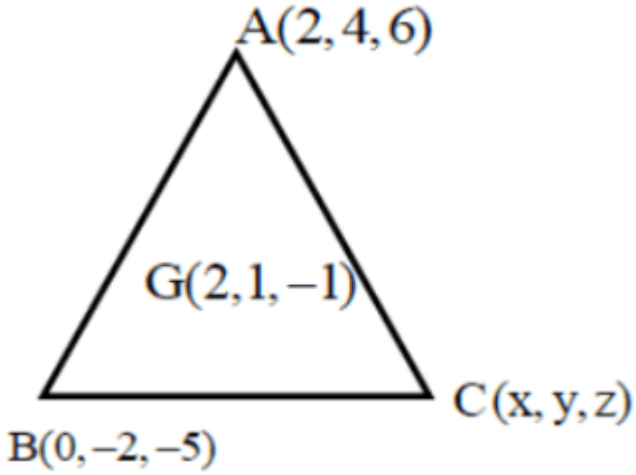
C. 70

D. 72

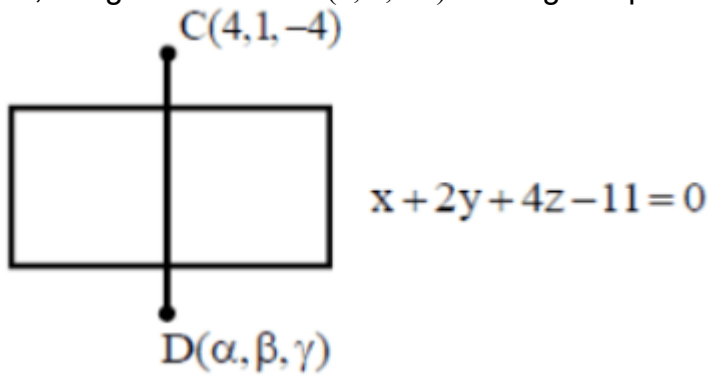
Answer: B

Solution:

Solution:



Given Two vertices of Triangle $A(2, 4, 6)$ and $B(0, -2, -5)$ and if centroid $G(2, 1, -1)$
Let Third vertices be (x, y, z)
Now $\frac{2+0+x}{3} = 2,\ \frac{4-2+y}{3} = 1,\ \frac{6-5+z}{3} = -1$
$x = 4, y = 1,\quad z = -1$
Third vertices $C(4, 1, -4)$
Now, Image of vertices $C(4, 1, -4)$ in the given plane is $D(\alpha, \beta, \gamma)$



Now
$\frac{\alpha-4}{1} = \frac{\beta-1}{2} = \frac{\gamma+4}{4} = -2\,\frac{(4+2-16-11)}{1+4+16}$
$\frac{\alpha-4}{1} = \frac{\beta-1}{2} = \frac{\gamma+4}{4} = \frac{42}{21} \Rightarrow 2$
$\alpha = 6, \beta = 5, \gamma = 4$
Then $\alpha\beta + \beta\gamma + \gamma\alpha$
$= (6 \times 5) + (5 \times 4) + (4 \times 6)$
$= 30 + 20 + 24$
$= 74$

Question108

The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$ is :
[10-Apr-2023 shift 1]

Options:

A. 8

B. 7

C. 6

D. 9

Answer: D

Solution:

Solution:
$\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$

$$= \frac{|-54|}{\left|-4\hat{i}+2\hat{j}+4k\right|}$$

$$= \frac{54}{\sqrt{16+4+16}}$$

$$= \frac{54}{6}$$

$$= 9$$

Question109

Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane x + y + z = 2. If the distance of the point P from the plane 3x − 4y + 12z = 32 is q, then q and 2q are the roots of the equation :
[10-Apr-2023 shift 1]

Options:

A. $x^2 + 18x - 72 = 0$

B. $x^2 + 18x + 72 = 0$

C. $x^2 - 18x - 72 = 0$

D. $x^2 - 18x + 72 = 0$

Answer: D

Solution:

Solution:

$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$

$x = 3\lambda - 3, y = \lambda - 2, z = 1 - 2\lambda$

$P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of plane $x + y + z = 2$.

$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$

$2\lambda - 4 = 2$

$\lambda = 3$

$P(6, 1, -5)$

Perpendicular distance of P from plane $3x - 4y + 12z - 32 = 0$ is

$q = \left| \frac{3(6) - 4(1) + 12(-5) - 32}{\sqrt{9 + 16 + 144}} \right|$

$q = 6$

$2q = 12$

Sum of roots $= 6 + 12 = 18$

Product of roots $= 6(12) = 72$

∴ Quadratic equation having q and $2q$ as roots is $x^2 - 18x + 72$.

Question110

Let time image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be Q(α, β, γ). Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to [10-Apr-2023 shift 2]

Options:

A. 70

B. 76

C. 62

D. 65

Answer: D

Solution:

Solution:
Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$
Put $(1, 4, 1) \Rightarrow 2B+C=0$
Put $(0, 5, 1) \Rightarrow -A+3B+C=0$
Sub : $B-A=0 \Rightarrow A=B, C=-2B$
$1(x-1)+1(y-2)-2(z-0)=0$
$x+y-2z-3=0$
Image is $(\alpha, \beta, \gamma)pt \equiv (1, 2, 6)$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$$

$\alpha=5, \beta=6, \gamma=-2 \Rightarrow \alpha^2+\beta^2+\gamma^2$
$=25+36+4=65$

Question111

Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x-2y+z=14$ is
[10-Apr-2023 shift 2]

Options:

A. 3

B. $\frac{10}{3}$

C. 4

D. $\frac{11}{3}$

Answer: C

Solution:

Solution:

$$\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda \ldots (1)$$

$$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu \ldots (2)$$

$\frac{x+3}{6} = \frac{y-3}{-3} = \frac{z-6}{1} = \gamma$ . . . (3)

Intersection of (1)&(2) "A"

$(\lambda, -2\lambda+6, 5\lambda-8)\&(4\mu+5, 3\mu+7, \mu-2)$

$\lambda = 1, \mu = -1$

A(1, 4, −3)

Intersection (1) & (3) B" "

$(\lambda, -2\lambda+6, 5\lambda-8)\&(6\gamma-3, -3\gamma+3, \gamma+6)$

$\lambda = 3$

$\gamma = 1$

B(3, 0, 7)

Mod point of A & B $\Rightarrow$ (2, 2, 2)

Perpendicular distance from the plane

$2x - 2y + z = 14$

$\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right| = 4$

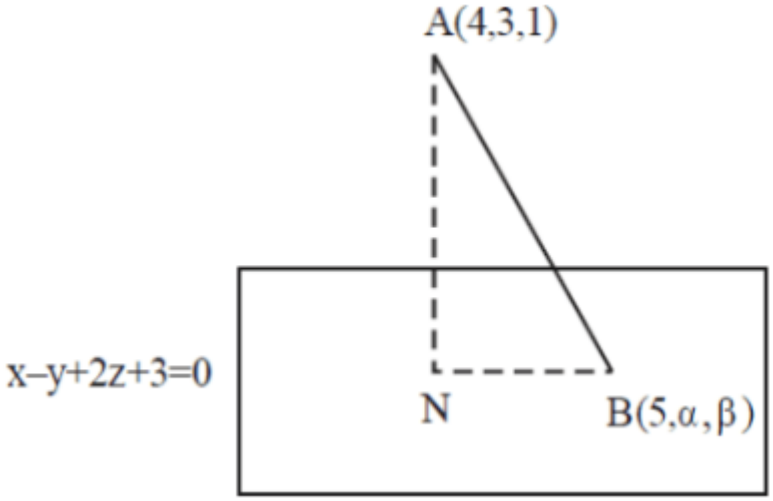
Question112

Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : x − y + 2z + 3 = 0 be N. If B(5, α, β), α, β ∈ Z is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _____.
[10-Apr-2023 shift 2]

Answer: 7

Solution:

Solution:



$AN = \sqrt{6}$

$5 - \alpha + 2\beta + 3 = 0$

$\Rightarrow \alpha = 8 + 2\beta$

N is given by

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$$

$x = 3, y = 4, z = -1$

N

$(3, 4, -1)$

$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$

$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$

Area of $\triangle ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$

$\frac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2}$

$BN = 2\sqrt{3}$

$4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$

$(2\beta + 4)^2 + (\beta + 1)^2 - 8 = 0$

$5\beta^2 + 18\beta + 9 = 0$

$(5\beta + 3)(\beta + 3) = 0$

$\beta = -3$

$\alpha = 2$

$\alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$

Question113

Let (α, β, γ) be the image of the point P(2, 3, 5) in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to : [11-Apr-2023 shift 1]

Options:

A. 5

B. 9

C. 10

D. 12

Answer: C

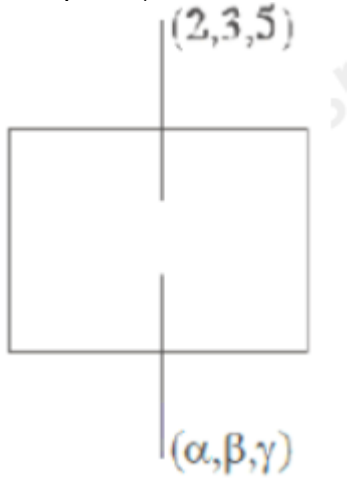
Solution:

Solution:

$$\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-2\left(\frac{2\times2+3-3\times5-6}{2^2+1^2+1-3^2}\right)=2$$

$$\frac{\alpha-2}{2}=2\ \beta-3=2\ \gamma-5=-6$$

$\alpha=6\ \beta=5\ \gamma=-1$



$\alpha+\beta+\gamma=10$

Question114

If the equation of the plane that contains the point (−2, 3, 5) and is perpendicular to each of the planes $2x+4y+5z=8$ and $3x-2y+3z=5$ is $\alpha x+\beta y+\gamma z+97=0$ then $\alpha+\beta+\gamma=$: [11-Apr-2023 shift 1]

Options:

A. 15

B. 18

C. 17

D. 16

Answer: A

Solution:

Solution:
The equation of plane through $(-2, 3, 5)$ is
$a(x+2)+b(y-3)+c(z-5)=0$
it is perpendicular to $2x+4y+5z=8\&3x-2y+3z=5$
$\therefore 2a+4b+5c=0$
$3a-2b+3c=0$

$$\therefore \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$$

∴ Equation of plane is

$22(x+2)+9(y-3)-16(z-5)=0$

$\Rightarrow 22x+9y-16z+97=0$

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

Question115

Let a line l pass through the origin and be perpendicular to the lines $l_1 : \vec{r} = \hat{i} - 11\hat{j} - 7\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k}$, $\lambda \in \mathbb{R}$ and $l_2 : \vec{r} = -\hat{i} + \hat{k} + \mu 2\hat{i} + 2\hat{j} + \hat{k}$, $\mu \in \mathbb{R}$. If P is the point of intersection of l and l_1, and Q(α, β, γ) is the foot of perpendicular from P on l_2, then $9(\alpha+\beta+\gamma)$ is equal to _____. [11-Apr-2023 shift 1]

Answer: 5

Solution:

Solution:

Let $\ell = (0\hat{i}+0\hat{j}+0\hat{k}) + \gamma(a\hat{i}+b\hat{j}+c\hat{k})$

$= \gamma(a\hat{i}+b\hat{j}+c\hat{k})$

$$a\hat{i}+b\hat{j}+c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6)-\hat{j}(1-6)+\hat{k}(2-4)$

$= -4\hat{i}-5\hat{j}-2\hat{k}$

$\ell = \gamma(-4\hat{i}+5\hat{j}-2\hat{k})$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1+\lambda,\ 5\gamma = -11+2\lambda,\ -2\gamma = -7+3\lambda$

By solving these equation $\gamma = -1$, P(4, −5, 2)

Let $Q(-1+2\mu, 2\mu, 1+\mu)$

$\overrightarrow{PQ} \cdot \left(2\hat{i}+2\hat{j}+\hat{k}\right) = 0$

$-2+4\mu+4\mu+1+\mu = 0$

$9\mu = 1$

$\mu = \frac{1}{9}$

$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha+\beta+\gamma) = 9\left(\frac{-7}{9}+\frac{2}{9}+\frac{10}{9}\right)$

$= 5$

Question116

Let P be the plane passing through the points (5, 3, 0), (13, 3, −2) and (1, 6, 2). For α ∈ N, if the distances of the points A(3, 4, α) and B(2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is
[11-Apr-2023 shift 2]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

Normal of the plane $= 3\hat{i} - 4\hat{j} + 12\hat{k}$

Plane : $3x - 4y + 12z = 3$

Distance from $A(3, 4, \alpha)$

$$\left|\frac{9-16+12\alpha-3}{13}\right| = 2$$

$\alpha = 3$

$\alpha = -8$ (rejected)

Distance from $B(2, 3, a)$

$$\left| \frac{6 - 12 + 12a - 3}{13} \right| = 3$$

$a = 4$

Question117

Let the line passing through the point P(2, −1, 2) and Q(5, 3, 4) meet the plane $x - y + z = 4$ at the point T . Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

[11-Apr-2023 shift 2]

Options:

A. 3

B. $\sqrt{61}$

C. $\sqrt{31}$

D. $\sqrt{189}$

Answer: A

Solution:

Solution:

Line: $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \quad \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

Line: $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$

Point $T = (2\mu - 1, 2\mu - 5, \mu)$

It lies on plane

$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$

$\mu = 1$

$\therefore T = (1, -3, 1)$

$\therefore RT = 3$

Question118

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in R$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to _____.

[11-Apr-2023 shift 2]

Answer: 11

Solution:

Solution:

$\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$

Dr's of line $\ell(1, 2, \lambda)$

Dr's of normal vector of plane $P : x + 2y + 3z = 4$ are $(1, 2, 3)$

Now, angle between line ℓ and plane P is given by

$$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{given } \cos\theta = \sqrt{\frac{5}{14}}\right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$

line of plane P.

$$\Rightarrow t = -1$$

$$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha + 2\beta + 6\gamma = 11$$

Question119

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(−4, −3, 2), then $|a| + |b| + |c|$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 10

B. 8

C. 12

D. 14

Answer: A

Solution:

Solution:

$(3x+2y+z-2)+\mu(x-3y+2z-13)=0$

$3(3+\mu)+1\cdot(2-3\mu)-2(1+2\mu)=0$

$9-4\mu=0$

$\mu=\frac{9}{4}$

$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$

$-100+4\alpha-54+18\alpha=0$

$\Rightarrow \alpha=7$

Let $P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$

Direction ratio of PQ $(3\lambda-1,\lambda-1,-2\lambda+5)$

But $PQ\perp \ell_1$

$\Rightarrow 3(3\lambda-1)+1\cdot(\lambda-1)-2(-2\lambda+5)=0$

$\Rightarrow \lambda=1$

$P(-2,-3,5)\Rightarrow |a|+|b|+|c| = 10$

Question120

Let the plane P: $4x-y+z=10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x+y-z=4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to
[12-Apr-2023 shift 1]

Options:

A. 90

B. 105

C. 85

D. 126

Answer: D

Solution:

Solution:
Let equation in new position is
$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$
$4(4 + \lambda) - 1 \cdot (-1 + \lambda) + 1 \cdot (1 - \lambda) = 0$
$\Rightarrow \lambda = -9$
So equation in new position is
$-5x - 10y + 10z + 26 = 0$
$\Rightarrow \alpha = \frac{54}{15}$

Question121

Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in Z$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to
[13-Apr-2023 shift 1]

Options:

A. -4

B. 2

C. -2

D. 4

Answer: A

Solution:

Solution:
$(x + 2y + az - 2) + \lambda(x - y + z - 3) = 0$
$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$
$\lambda = -\frac{7}{2}, a = 3, b = 1$

$$\frac{2}{\sqrt{a}} = \left| \frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}} \right|$$

$c = -1$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

Question122

The distance of the point (−1, 2, 3) from the plane $\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) = 10$ parallel to the line of the shortest distance between the lines $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$ is
[13-Apr-2023 shift 1]

Options:

A. $2\sqrt{5}$

B. $3\sqrt{5}$

C. $3\sqrt{6}$

D. $2\sqrt{6}$

Answer: D

Solution:

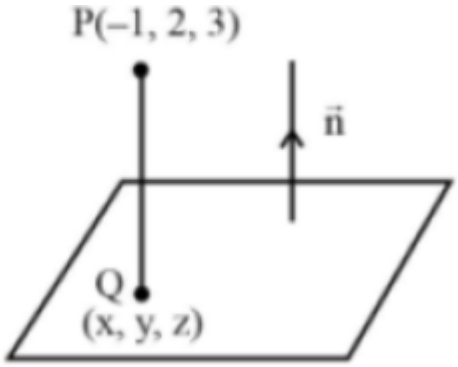
Solution:

Let $L_1 : \vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$

$L_2 : \vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$\vec{n} = \hat{i} - \hat{j} - 2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = r$

$\Rightarrow (x, y, z) \equiv (r-1, 2-r, 3-2r)$

$\Rightarrow (r-1) - 2(2-r) + 3(3-2r) = 10$

$\Rightarrow r = -2$

$\Rightarrow Q(x, y, z) \equiv (-3, 4, 7)$

$\Rightarrow PQ = \sqrt{4+4+16} = 2\sqrt{6}$

Question123

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha)$, $\alpha > 0$, from P is 13 , then α is equal to _____.
[13-Apr-2023 shift 1]

Answer: 15

Solution:

Solution:

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3} + (-2)\times\frac{8}{3} + 1\times\frac{8}{3} - 2\right)}{1^2+2^2+1^2}$$

$$= \frac{1}{3}$$

$\therefore x = 2, y = 1, z = 3$

$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$

$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

Question124

The plane, passing through the points (0, −1, 2) and (−1, 2, 1) and parallel to the line passing through (5, 1, −7) and (1, −1, −1), also passes through the point
[13-Apr-2023 shift 2]

Options:

A. $(0, 5, -2)$

B. $(-2, 5, 0)$

C. $(2, 0, 1)$

D. $(1, -2, 1)$

Answer: B

Solution:

Solution:
Plane passing through $(0, -1, 0)$ and $(-1, 2, 1)$
Then vector in plane $\langle -1, 3, -1\rangle$ vector parallel to plane is $\langle 4, 2, -6\rangle$

Normal vector to plane $(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$
$\vec{n} = \langle 8, 5, 7\rangle$
Equation of plane
$8(x-0) + 5(y+1) + 7(z-2) = 0$
$\Rightarrow 8x + 5y + 7z = 9$
From given options point $(-2, 5, 0)$ lies on plane.

Question125

The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is
[13-Apr-2023 shift 2]

Options:

A. $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

B. $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

C. $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

D. $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

Answer: A

Solution:

Solution:
Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a1, b1, c1 are direction cosine of 1^{st} line and a2, b2, c2 are direction cosine of 2^{nd} line.
Now. Solving options
Point $(-3, 1, 5)$& point $(-1, 2, 5)$

$$(1)\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$$= -3(5) - (10) + 5(-1 + 4)$$
$$= -15 - 10 + 15 = -10$$

(2) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$$= 3(5) - (10) + 5(1 + 4)$$
$$-25 + 5 = 0$$

(3) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

$$-3(4) - (8) + 5(1 + 4)$$
$$-12 - 8 + 25 = 5$$

(4) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$-3(-5)-(-20)+5(-1-8)$
$15+20-45=-10$

Question126

Let N be the foot of perpendicular from the point P(1, −2, 3) on the line passing through the points (4, 5, 8) and (1, −7.5). Then the distance of N from the plane $2x-2y+z+5=0$ is [13-Apr-2023 shift 2]

Options:

A. 6

B. 7

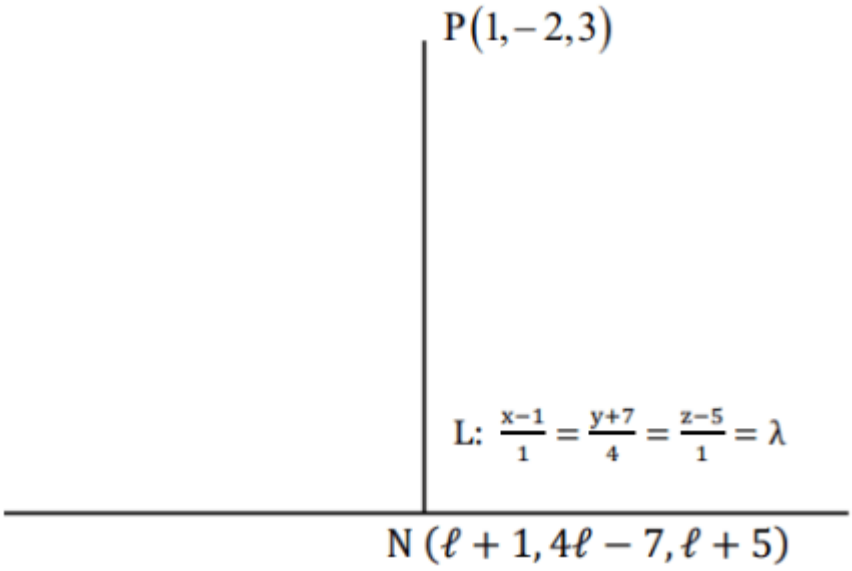
C. 9

D. 8

Answer: B

Solution:

Solution:



$\overrightarrow{PN}=\langle\lambda, 4\lambda-5, \lambda+2\rangle$
$\overrightarrow{PN}.<1, 4, 1>=0$
$\Rightarrow \lambda+16\lambda-20+\lambda+2=0$
$\Rightarrow \lambda=1$
N(2, −3, 6)
Distance of N from $2x-2y+z+5=0$ is

$$d = \left| \frac{2(2) - 2(-3) + 6 + 5}{\sqrt{2^2 + (-2)^2 + (1)^2}} \right|$$

$$= \left| \frac{21}{3} \right| = 7$$

Question127

Let the foot of perpendicular of the point P(3, −2, −9) on the plane passing through the points (−1, −2, −3), (9, 3, 4), (9, −2, 1) be Q(α, β, γ). Then the distance of Q from the origin is [15-Apr-2023 shift 1]

Options:

A. $\sqrt{29}$

B. $\sqrt{38}$

C. $\sqrt{42}$

D. $\sqrt{35}$

Answer: C

Solution:

Solution:
P(3, −2, −9)
Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$$2x + 3y - 5z - 7 = 0$$

Foot of I^r of P(3, −2, −9) is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(6-6+45-7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$$Q(1, -5, -4) \equiv (\alpha, \beta, \gamma)$$

$$OQ = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{42}$$

Question128

Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13 . Then $8\left|\sum_{\lambda \in S} \lambda\right|$ is equal to

[15-Apr-2023 shift 1]

Options:

A. 302

B. 306

C. 304

D. 308

Answer: B

Solution:

Solution:

$$\text{Short test distance} = \frac{\begin{vmatrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2\lambda & 3 & -12 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}}$$

$$13 = \frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$$

$$= \frac{|153+8\lambda|}{13}$$

$$|153+8\lambda| = 169$$

$$153+8\lambda = 169, -169$$

$$\lambda = \frac{16}{8}, \frac{-322}{8}$$

$$8\left|\sum_{\lambda \in S} \lambda\right| = 306$$

Question129

If the line $x = y = z$ intersects the line $x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to _____

[15-Apr-2023 shift 1]

Answer: 5

Solution:

Solution:

$\sin A + \sin B + \sin C = \frac{18}{x}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$

$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

$4\cos A/2\cos B/2\cos C/2 = 2(4\sin A\sin B\sin C)$

$16\sin A/2\sin B/2\sin C/2 = 1$

Hence Ans. $= 5$.

Question130

Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$ Then the distance of the point A(8, −1, −19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _____

[15-Apr-2023 shift 1]

Answer: 26

Solution:

Solution:

Plane $\equiv P_1 = \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

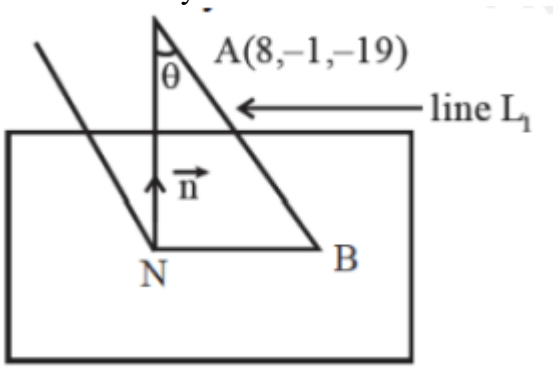
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n} \cdot \vec{b} = 0$ where $\vec{b}(2, 4, 5)$

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$

Let $B = (8 - 3\lambda, -1 + 4\lambda, -19 + 12\lambda)$ lies on plane P

$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$

$\lambda = 2$

$\therefore$ Point $B = (2, 7, 5)$

$AB = \sqrt{6^2 + 8^2 + 24^2} = 26$

Question131

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible value of λ is : [24-Jun-2022-Shift-2]

Options:

A. 16

B. 6

C. 12

D. 15

Answer: A

Solution:

Solution:

Let $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}, \vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$

$\therefore \vec{p} \times \vec{q} = (15 - 4\lambda)\hat{i} - (10 - \lambda)\hat{j} + 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ Shortest distance

$$= \left| \frac{(15 - 4\lambda) - 2(10 - \lambda) + 10}{\sqrt{(15 - 4\lambda)^2 + (10 - \lambda)^2 + 25}} \right| = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 3(5 - 2\lambda)^2 = (15 - 4\lambda)^2 + (10 - \lambda)^2 + 25$$

$$\Rightarrow 5\lambda^2 - 80\lambda + 275 = 0$$

$\therefore$ Sum of values of $\lambda = \frac{80}{5} = 16$

Question132

Let the points on the plane P be equidistant from the points (−4, 2, 1) and (2, −2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is [24-Jun-2022-Shift-2]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: C

Solution:

Solution:

Let $P(x, y, z)$ be any point on plane P_1

Then $(x-4)^2-(y-2)^2+(z-1)^2=(x-2)^2+(y+2)^2+(z-3)^2$

$\Rightarrow 12x-8y+4z+4=0$

$\Rightarrow 3x-2y+z+1=0$

And $P_2: 2x+y+3z=0$

$\therefore$ angle between P_1 and P_2

$\cos\theta\left|\frac{6-2+3}{14}\right| \Rightarrow \theta=\frac{\pi}{3}$

Question133

Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, −1, −1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to : [25-Jun-2022-Shift-1]

Options:

A. 2

B. 5

C. 7

D. 11

Answer: B

Solution:

Solution:

As L is parallel to PQ d.r.s of S is $\langle 1, 1, 1\rangle$

$$\therefore L \equiv \frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$$

Point of intersection of L and S be λ

$$\Rightarrow (\lambda+1)+(\lambda-1)+(\lambda-1) = S$$

$$\Rightarrow \lambda = 2$$

$$\therefore R \equiv (3, 1, 1)$$

Let $Q(\alpha, \beta, \gamma)$

$$\Rightarrow \frac{\alpha-1}{1} = \frac{\beta}{1} = \frac{\gamma-1}{1} = \frac{-2(-3)}{3}$$

$$\Rightarrow \alpha = 3, \beta = 2, \gamma = 3$$

$$\Rightarrow Q \equiv (3, 2, 3)$$

$$(QR)^2 = 0^2 + (1)^2 + (2)^2 = 5$$

Question134

Let the lines

$$\mathbf{L_1 : \vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\,widehat\,k\right), \lambda \in R}$$

$$\mathbf{L_2 : \vec{r} = \left(\hat{i} + 3\hat{j} + widehat\,k\right) + \mu\left(\hat{i} + \hat{j} + 5\,widehat\,k\right); \mu \in R}$$

intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2, then the value of $a + b + d$ is equal to

[25-Jun-2022-Shift-1]

Answer: 5

Solution:

Solution:

As plane is parallel to both the lines we have d.r's of normal to the plane as <7, −2, −1>

$$\left(\text{from} \begin{vmatrix} \hat{i} & \hat{j} & \text{widehat } k \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - \hat{j}(2) + \text{widehat } k(-1) \right)$$

Also point of intersection of lines is $2\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore$ Equation of plane is

$7(x-2) - 2(y-4) - 1(z-6) = 0$

$\Rightarrow 7x - 2y - z = 0$

$a + b + d = 7 - 2 + 0 = 5$

Question135

Let p be the plane passing through the intersection of the planes $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 5$ and $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$, and the point (2, 1, −2). Let the position vectors of the points X and Y be $\hat{i} - 2\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 2\hat{k}$ respectively. Then the points [25-Jun-2022-Shift-2]

Options:

A. X and X + Y are on the same side of P

B. Y and Y − X are on the opposite sides of P

C. X and Y are on the opposite sides of P

D. X + Y and X − Y are on the same side of P

Answer: C

Solution:

Solution:
Let the equation of required plane
$\pi : (x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$
$\because$(2, 1, −2) lies on it so, $2 + \lambda(-2) = 0$
$\Rightarrow \lambda = 1$
Hence, $\pi : 3x + 2y - 8 = 0$

$\because \pi_x = -9, \pi_y = 5, \pi_{x+y} = 4$
$\pi_{x-y} = -22$ and $\pi_{y-x} = 6$
Clearly X and Y are on opposite sides of plane π

Question136

Let I_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and I_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line I_1 and I_2, then d^{-2} is equal to____
[25-Jun-2022-Shift-2]

Answer: 51

Solution:

Solution:

$$\frac{x-\frac{1}{8}}{\frac{1}{8}} = \frac{y}{-\frac{1}{4\sqrt{2}}} = \frac{z}{0} \quad L_1$$

or $\frac{x-\frac{1}{8}}{1} = \frac{y}{-\sqrt{2}} = \frac{z}{0}$..... (i)

Equation of L_2

$$\frac{x+\frac{1}{8}}{-6\sqrt{3}} = \frac{y}{0} = \frac{z}{8}$$..... (ii)

$$d = \left| \frac{(c-\vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$$

$$= \frac{\left(\frac{1}{4}\hat{i}\right) \cdot \left(4\sqrt{2}\hat{i} + 4\hat{j} + 3\sqrt{6}\hat{k}\right)}{\sqrt{(4\sqrt{2})^2 + 4^2 + (3\sqrt{6})^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{32+16+54}} = \frac{1}{\sqrt{51}}$$

$$d^{-2} = 51$$

Question137

If the two lines $l_1: \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2: \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ are perpendicular, then an angle between the lines I_2 and $l_3: \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :
[26-Jun-2022-Shift-1]

Options:

A. $\cos^{-1}\left(\frac{29}{4}\right)$

B. $\sec^{-1}\left(\frac{29}{4}\right)$

C. $\cos^{-1}\left(\frac{2}{29}\right)$

D. $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Answer: B

Solution:

Solution:

$\because L_1$ and L_2 are perpendicular, so

$$3\times 1+(-2)\left(\frac{\alpha}{2}\right)+0\times 2=0$$

$$\Rightarrow \alpha = 3$$

Now angle between I_2 and I_3,

$$\cos\theta = \frac{1(-3)+\frac{\alpha}{2}(-2)+2(4)}{\sqrt{1-\frac{\alpha^2}{4}+4}\sqrt{9+4+16}}$$

$$\Rightarrow \cos\theta = \frac{2}{\frac{29}{2}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{29}\right) = \sec^{-1}\left(\frac{29}{4}\right)$$

Question138

Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is B(a, b, c), then :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

B. $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$

C. $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$

D. $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

Answer: A

Solution:

Solution:

Consider the equation of plane,

$P : (2x + 3y + z + 20) + \lambda(x - 3y - 5z - 8) = 0$

$P : (2 + \lambda)x + 3(3 - 3\lambda)y - 1(1 + 5\lambda)z + (20 - 8\lambda) = 0$

$\because$ Plane P is perpendicular to $2x - 3y + z + 20 = 0$

So, $4 - 2\lambda + 9 - 9\lambda + 1 + 5\lambda = 0$

$\Rightarrow \lambda = 7$

$P : 9x - 18y + 36z - 36 = 0$

or $P : x - 2y + 4z = 4$

If image of $\left(2, -\frac{1}{2}, 2\right)$ in plane P is (a, b, c) then

$$\frac{a-2}{1} = \frac{b+\frac{1}{2}}{-2} = \frac{c-2}{4}$$

and $\left(\frac{a+2}{2}\right) - 2\left(\frac{b-\frac{1}{2}}{2}\right) - 4\left(\frac{c-2}{2}\right) = 4$

Clearly $a = \frac{4}{3}$, $b = \frac{5}{6}$ and $c = -\frac{2}{3}$

So, $a : b : c = 8 : 5 : -4$

Question139

If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point : [26-Jun-2022-Shift-2]

Options:

A. $(2, -2, 0)$

B. $(-2, 2, 0)$

C. $(1, 0, 2)$

D. $(-1, 0, -2)$

Answer: C

Solution:

Solution:

$P_1 : 2x + y - 52 = 0, P_2 : 3x - y + 4z - 7 = 0$

Family of planes P_1 and P_2

$P : P_1 + \lambda P_2$

$\therefore P : (2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$

$\because P \perp P_1$

$\therefore 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$

$\lambda = 2$

$\therefore P : 8x - y + 32 - 14 = 0$

It passes through the point (1, 0, 2)

Question140

If the lines $\vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(\alpha\hat{i} - \hat{j}\right) + \mu\left(2\hat{i} - 3\hat{k}\right)$ are co-planar, then the distance of the plane containing these two lines from the point (α, 0, 0) is :
[26-Jun-2022-Shift-2]

Options:

A. $\frac{2}{9}$

B. $\frac{2}{11}$

C. $\frac{4}{11}$

D. 2

Answer: B

Solution:

Solution:

$\because$ Both lines are coplanar, so

$$\begin{vmatrix} \alpha-1 & 0 & -1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} =$$

$$\Rightarrow \alpha = \frac{5}{3}$$

Equation of plane containing both lines

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 9x+2y+6z = 13$$

So, distance of $\left(\frac{5}{3}, 0, 0\right)$ from this plane

$$= \frac{2}{\sqrt{81+4+36}} = \frac{2}{11}$$

Question141

If two straight lines whose direction cosines are given by the relations $l + m - n = 0, 3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is : [27-Jun-2022-Shift-1]

Options:

A. 6

B. 4

C. 3

D. 2

Answer: A

Solution:

Solution:
$l + m - n = 0 \Rightarrow n = l + m$
$3l^2 + m^2 + cnl = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$= (3 + c)l^2 + cl\,m + m^2 = 0$

$= (3 + c)\left(\frac{1}{m}\right)^2 + c\left(\frac{1}{m}\right) + 1 = 0$

$\therefore$ Lines are parallel

$D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 4)(c + 3) = 0$

$c = 4$(as $c > 0$)

Question142

Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to
[27-Jun-2022-Shift-1]

Answer: 137

Solution:

Solution:

$$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-2(3a-4b+12c+19)}{3^2+(-4)^2+12^2}$$

$$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-6a+8b-24c-38}{169}$$

$(x, y, z) \equiv (a - 6, \beta, \gamma)$

$$\frac{(a-6)-a}{3} = \frac{\beta-b}{-4} = \frac{\gamma-c}{12} = \frac{-6a+8b-24c-38}{169}$$

$$\frac{\beta-b}{-4} = -2 \Rightarrow \beta = 8 + b$$

$$\frac{\gamma-c}{12} = -2 \Rightarrow \gamma = -24 + c$$

$$\frac{-6a+8b-24c-38}{169} = -2$$

$\Rightarrow 3a - 4b + 12c = 150$

$a + b + c = 5$

$3a + 3b + 3c = 15$
Applying (1) - (2)
$-7b + 9c = 135$
$7b - 9c = -135$
$7\beta - 9\gamma = 7(8 + b) - 9(-24 + c)$
$= 56 + 216 + 7b - 9c$
$= 56 + 216 - 135 = 137$

Question143

Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is [27-Jun-2022-Shift-2]

Options:

A. 5

B. $\frac{50}{13}$

C. 4

D. $\frac{63}{13}$

Answer: A

Solution:

Solution:
$L: \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = t$
Let $P = (4t - 2, 2t + 1, 3t - 1)$
$\because P$ is the foot of perpendicular of $(1, 2, 4)$
$\therefore 4(4t - 3) + 2(2t - 1) + 3(3t - 5) = 0$
$\Rightarrow 29t = 29 \Rightarrow t = 1$
$\therefore P = (2, 3, 2)$
Now, distance of P from the plane
$3x + 4y + 12z + 23 = 0$, is
$\left| \frac{6 + 12 + 24 + 23}{\sqrt{9 + 16 + 144}} \right| = \frac{65}{13} = 5$

Question144

The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$, is
[27-Jun-2022-Shift-2]

Options:

A. $\frac{18}{\sqrt{5}}$

B. $\frac{22}{3\sqrt{5}}$

C. $\frac{46}{3\sqrt{5}}$

D. $6\sqrt{3}$

Answer: A

Solution:

Solution:

$L_1: \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$

$L_2: \frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$

Now, $\vec{p}\times\vec{q}=\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 2 & 1 & 3\end{array}\right|=10\hat{i}-8\hat{j}-4\hat{k}$

and $\vec{a}_2-\vec{a}_1=6\hat{i}-4\hat{j}-4\hat{k}$

$\therefore S.D.=\left|\frac{60+32+16}{\sqrt{100+64+16}}\right|=\frac{108}{\sqrt{180}}=\frac{18}{\sqrt{5}}$

Question145

If two distinct point Q, R lie on the line of intersection of the planes $-x+2y-z=0$ and $3x-5y+2z=0$ and $PQ=PR=\sqrt{18}$ where the point P is $(1,-2,3)$, then the area of the triangle PQR is equal to
[28-Jun-2022-Shift-1]

Options:

A. $\frac{2}{3}\sqrt{38}$

B. $\frac{4}{3}\sqrt{38}$

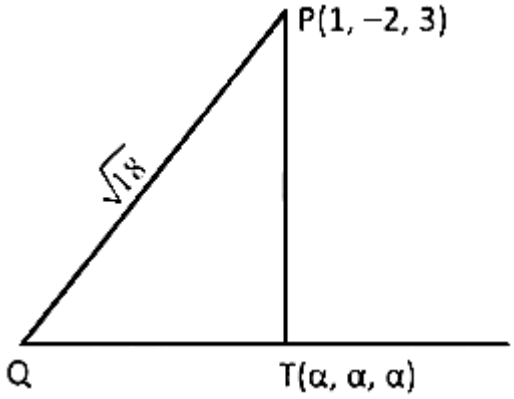
C. $\frac{8}{3}\sqrt{38}$

D. $\sqrt{\frac{152}{3}}$

Answer: B

Solution:

Solution:



Line L is $x = y = z$

$\overrightarrow{PQ} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\Rightarrow (\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$

$\Rightarrow \alpha = \frac{2}{3}$ so, $T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$PT = \sqrt{\frac{38}{3}}$

$\Rightarrow QT = \frac{4}{\sqrt{3}}$

So, Area $= \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}}\right) \cdot 2$

$= \frac{4\sqrt{38}}{3}$ sq. units

Question146

The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is
[28-Jun-2022-Shift-1]

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer: A

Solution:

Solution:
Family of Plane's equation can be given by
$(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z - (29 + 20\lambda) = 0$
P_1 passes through $(2, 1, 3)$
$\Rightarrow (10 + 16\lambda) + (8 - 7\lambda) + (39 + 3\lambda) - (29 + 20\lambda) = 0$
$\Rightarrow -8\lambda + 28 = 0 \Rightarrow \lambda = \frac{7}{2}$
d.r, s of normal to P_1
$\left\langle 33, \frac{-33}{2}, \frac{33}{2} \right\rangle$ or $\left\langle 1, -\frac{1}{2}, \frac{1}{2} \right\rangle$
P_2 passes through $(0, 1, 2)$
$\Rightarrow 8 - 7\lambda + 26 + 2\lambda - (29 + 20\lambda) = 0$
$\Rightarrow 5 - 25\lambda = 0$
$\Rightarrow \lambda = \frac{1}{5}$
d.r, s of normal to P_2
$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5} \right\rangle$ or $\langle 1, 1, 2 \rangle$
Angle between normals

$$= \frac{\left(\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right) \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right)}{\frac{\sqrt{3}}{2}}$$

$$\cos\theta = \frac{1-\frac{1}{2}+1}{3} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Question147

Let the plane P : $\vec{r}\cdot\vec{a}=d$ contain the line of intersection of two planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=6$ and $\vec{r}\cdot\left(-6\hat{i}+5\hat{j}-\hat{k}\right)=7$ If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. 90

B. 93

C. 95

D. 97

Answer: B

Solution:

Solution:
$P_1 : x+3y-z=6$
$P_2 : -6x+5y-z=7$
Family of planes passing through line of intersection of P_1 and P_2 is given by
$x(1-6\lambda)+y(3+5\lambda)+z(-1-\lambda)-(6+7\lambda)=0$
It passes through $\left(2, 3, \frac{1}{2}\right)$

$$\text{So, } 2(1-6\lambda)+3(3+5\lambda)+\frac{1}{2}(-1-\lambda)-(6+7\lambda)=0$$

$$\Rightarrow 2-12\lambda+9+15\lambda-\frac{1}{2}-\frac{\lambda}{2}-6-7\lambda=0$$

$$\Rightarrow \frac{9}{2}-\frac{9\lambda}{2}=0 \Rightarrow \lambda=1$$

Required plane is
$-5x+8y-2z-13=0$

Or $\vec{r} \cdot \left(-5\hat{i} + 8\hat{j} - 2\hat{k}\right) = 13$

$$\frac{|13\vec{a}|^2}{|d|^2} = \frac{13^2}{(13)^2} \cdot \left|\vec{a}\right|^2 = 93$$

Question148

Let the plane ax + by + cz = d pass through (2, 3, −5) and is perpendicular to the planes 2x + y − 5z = 10 and 3x + 5y − 7z = 12. If a, b, c, d are integers d > 0 and gcd (|a|, | b | , | c | ,d) = 1, then the value of a + 7b + c +20d is equal to : [28-Jun-2022-Shift-2]

Options:

A. 18

B. 20

C. 24

D. 22

Answer: D

Solution:

Solution:
Equation of pane through point $(2, 3, -5)$ and perpendicular to planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$\begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 0$$

$\therefore$ Equation of plane is $(x-2)(-7+25) - (y-3)$
$(-14+15) + (z+5) \cdot 7 = 0$
$\therefore 18x - y + 7z + 2 = 0$
$\Rightarrow 18x - y + 7z = -2$
$\therefore -18x + y - 7z = 2$
On comparing with $ax + by + cz = d$ where $d > 0$ is $a = -18, b = 1, c = -7, d = 2$
$\therefore a + 7b + c + 20d = 22$

Question149

Let the image of the point P(1, 2, 3) in the line L : $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. Let R(α, β, γ) be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to___
[28-Jun-2022-Shift-2]

Answer: 125

Solution:

Solution:
The point dividing PQ in the ratio $1:3$ will be mid-point of P& foot of perpendicular from P on the line.
$\therefore$ Let a point on line be λ

$$\Rightarrow \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

$$\Rightarrow P'(3\lambda+6, 2\lambda+1, 3\lambda+2)$$

as P' is foot of perpendicular

$$(3\lambda+5)3 + (2\lambda-1)2 + (3\lambda-1)3 = 0$$

$$\Rightarrow 22\lambda + 15 - 2 - 3 = 0$$

$$\Rightarrow \lambda = \frac{-5}{11}$$

$$\therefore P'\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$$

$$\text{Mid-point of } PP' \equiv \left(\frac{\frac{51}{11}+1}{2}, \frac{\frac{1}{11}+2}{2}, \frac{\frac{7}{11}+3}{2}\right)$$

$$\equiv \left(\frac{62}{22}, \frac{23}{22}, \frac{40}{22}\right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow 22(\alpha, \beta, \gamma) = 62 + 23 + 40 = 125$$

Question150

If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), then $2a + b + 2c$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 54

B. 50

C. −6

D. −42

Answer: C

Solution:

Solution:
We know mirror image of point (x_1, y_1, z_1) in the plane $ax + by + cz = d$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 - d)}{a^2 + b^2 + c^2}$$

Here given point $(2, 4, 7)$ and plane $3x - y + 4z = 2$ then mirror image is

$$\frac{x - 2}{3} = \frac{y - 4}{-1} = \frac{z - 7}{4} = \frac{-2(6 - 4 + 28 - 2)}{9 + 1 + 16}$$

$$\Rightarrow \frac{x - 2}{3} = \frac{y - 4}{-1} = \frac{z - 7}{4} = -\frac{28}{13}$$

$$\therefore x = -\frac{58}{13} = a$$

$$y = \frac{80}{13} = b$$

$$z = -\frac{21}{13} = c$$

$$\therefore 2a + b + 2c$$

$$= 2\left(-\frac{58}{13}\right) + \frac{80}{13} + 2\left(-\frac{21}{13}\right)$$

$$= \frac{-116 + 80 - 42}{13} = \frac{-78}{13} = -6$$

Question151

Let d be the distance between the foot of perpendiculars of the points P(1, 2, −1) and Q(2, −1, 3) on the plane −x + y + z = 1. Then d^2 is equal to___
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Solution:
Foot of perpendicular from P

$$\frac{x-1}{-1}=\frac{y-2}{1}=\frac{z+1}{1}=\frac{-(-1+2-1-1)}{3}$$

$$\Rightarrow p' \equiv \left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

and foot of perpendicular from Q

$$\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z-3}{1}=\frac{-(-2-1+3-1)}{3}$$

$$\Rightarrow Q' \equiv \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$$P'Q' = \sqrt{(1)^2+(3)^2+(4)^2} = d = \sqrt{26}$$

$$\Rightarrow d^2 = 26$$

Question152

Let $P_1 : \vec{r} \cdot \left(2\hat{i}+\hat{j}-3\hat{k}\right) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, −3, 2), (2, −2, −3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to____
[29-Jun-2022-Shift-1]

Answer: 28

Solution:

Solution:
Direction ratio of normal to $P_1 \equiv <2, 1, -3>$

and that of $P_2 \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -5 \\ -1 & -2 & 5 \end{vmatrix} = -5\hat{i} - \hat{j}(-5) + \hat{k}(1)$

i.e. $\langle -5, 5, 1>$
d.r's of line of intersection are along vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = \hat{i}(16) - \hat{j}(-13) + \hat{k}(15)$$

i.e. $<16, 13, 15>$

$\therefore \alpha + \beta = 13 + 15 = 28$

Question153

Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in R$. The shortest distance of the plane from the origin is : [29-Jun-2022-Shift-2]

Options:

A. $\sqrt{\frac{3}{109}}$

B. $\sqrt{\frac{5}{142}}$

C. $\frac{5}{\sqrt{71}}$

D. $\frac{1}{\sqrt{142}}$

Answer: B

Solution:

Solution:
$(2, -1, -3)$ satisfy the given plane.
So $2p + q = 8$
Also given line is perpendicular to normal plane so $3p + 2q - 1 = 0$
$\Rightarrow p = 15, q = -22$
Eq. of plane $15x - 22y + z - 5 = 0$
its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

Question154

Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right)$, $\lambda \in R$. Then, which of the following points lies on T ?
[29-Jun-2022-Shift-2]

Options:

A. (2, 1, 0)

B. (1, 2, 1)

C. (1, 2, 2)

D. (1, 3, 2)

Answer: B

Solution:

Solution:
Image of $P(1, 2, 1)$ in $x + 2y + 2z - 16 = 0$

is given by $Q(4, 8, 7)$

Eq. of plane $T = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x - z = 1$ so $B(1, 2, 1)$ lies on it.

Question155

Let a line having direction ratios 1, −4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to__
[24-Jun-2022-Shift-1]

Answer: 84

Solution:

Solution:

Let $A(3\lambda+7, -\lambda+1, \lambda-2)$ and $B(2\mu, 3\mu+7, \mu)$

So, DR's of $AB \propto 3\lambda-2\mu+7, -(\lambda+3\mu+6), \lambda-\mu-2$

Clearly $\frac{3\lambda-2\mu+7}{1} = \frac{\lambda+3\mu+6}{4} = \frac{\lambda-\mu-2}{2}$

$\Rightarrow 5\lambda-3\mu=-16$

And $\lambda-5\mu=10$

From (i) and (ii) we get $\lambda=-5, \mu=-3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$AB=\sqrt{4+64+16} \Rightarrow (AB)^2=84$

Question156

Let α be the angle between the lines whose direction cosines satisfy the equations $l+m-n=0$ and $l^2+m^2-n^2=0$. Then, the value of $\sin^4\alpha+\cos^4\alpha$ is
[25 Feb 2021 Shift 1]

Options:

A. $\frac{3}{4}$

B. $\frac{3}{8}$

C. $\frac{5}{8}$

D. $\frac{1}{2}$

Answer: C

Solution:

Solution:

Given, $l + m - n = 0$. . . (i)
and $l^2 + m^2 - n^2 = 0$ ... (ii)
On squaring Eq. (i), we get
$(l + m)^2 = n^2$
$\Rightarrow\ l^2 + m^2 + 2lm = n^2$ . . . (iii)
From Eqs. (ii) and (iii),
$l^2 + m^2 - n^2 = 0$
$l^2 + m^2 + 2lm = n^2$
$- - \quad - \quad -$
$-n^2 - 2lm = -n^2$
$\Rightarrow\ 2lm = 0 \Rightarrow lm = 0$
$\Rightarrow\ l = 0$ or $m = 0$
Case I When $l = 0$
$\Rightarrow\ 0 + m - n = 0$
$\Rightarrow\ m = n$
and $l^2 + m^2 + n^2 = 1$
$\Rightarrow\ m^2 + m^2 = 1$ [$\because n = m$ and $l = 0$]
$\Rightarrow\ m^2 = \frac{1}{2}$
$m = \pm \frac{1}{\sqrt{2}} = n$
$\therefore\ (l, m, n) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
Case II When $m = 0$
then, $l + m - n = 0$
$\Rightarrow\ l = n$ and $l^2 + m^2 + n^2 = 1$
[$\because n = l$ and $m = 0$]
$\Rightarrow\ l^2 + 0 + l^2 = 1$
$l = \pm \frac{1}{\sqrt{2}}$ [$\because n = l$ and $m = 0$]
$\Rightarrow \therefore\ (l, m, n) = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$
$\Rightarrow\ a = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $b = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$
Then $\cos\alpha = \frac{a \cdot b}{|a|\,|b|} = \frac{1}{2}$
$\therefore \sin\alpha = \pm \frac{\sqrt{3}}{2}$
Now, $\cos^4\alpha + \sin^4\alpha = \frac{1}{16} + \frac{9}{16} = \frac{10}{16} = \frac{5}{8}$

Question157

A line ' l ' passing through origin is perpendicular to the lines

$$\mathbf{l_1 : r = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}}$$

$$\mathbf{l_2 : r(3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}}$$

If the coordinates of the point in the first octant on ' I_2 ' at a distance of $\sqrt{17}$ from the point of intersection of ' l ' and ' l_1 ' are (a, b, c), then 18(a + b + c) is equal to ..... .
[25 Feb 2021 Shift 2]

Answer: 44

Solution:

Solution:

Let $L_1 \Rightarrow \frac{x-3}{1} = \frac{y-3}{2} = \frac{z-4}{2} = u$ (say)

$\Rightarrow$ Direction ratios of $L_1 = 1, 2, 2$

$L_2 \Rightarrow \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = v$ (say)

Direction ratios of $L_2 = 2, 2, 1$

Line L passing through origin is perpendicular to L_1 and L_2.

Hence, direction ratios of L is parallel to $(L_1 \times L_2)$.

$\Rightarrow (-2, 3, -2)$

Equation of $L \Rightarrow \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$ (say)

Solve L and L_1, we get

$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + 4)$

Gives, $\lambda = 1, \mu = -1$

So, intersection point $P(2, -3, 2)$.

Let $Q(2v + 3, 2v + 3, v + 2)$ be required point on L_2.

Now, $PQ = \sqrt{17}$ (given)

Now, $PQ = \sqrt{17}$ (given)

$PQ = \sqrt{(2v+1)^2 + (2v+6)^2 + (v)^2}$

$= \sqrt{17}$

$\Rightarrow (2v + 1)^2 + (2v + 6)^2 + v^2 = 17$ (squaring on both sides)

$\Rightarrow 9v^2 + 28v + 20 = 0$

On solving, we get $v = -2$ (rejected), $\frac{-10}{9}$ (accepted)

$\therefore Q$ is $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

$\therefore 18(a + b + c) = 18\left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9}\right) = 44$

Question158

The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is
[25 Feb 2021 Shift 1]

Options:

A. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

B. $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

C. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

D. $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Answer: D

Solution:

Solution:
Given, line $\Rightarrow \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$ (let)
Any point on this line is $B(2\lambda+1, 3\lambda-1, -2\lambda+1)$ and direction ratios of this line $= (2, 3, -2) = d_1$
Let given point be $A(0, 1, 2)$.
Then direction ratio of
$AB = (2\lambda+1, 3\lambda-2, -2\lambda-1) = d_2$
$\because$ Both lines are perpendicular to each other.
$\therefore \; d_1 \cdot d_2 = 0$
$2(2\lambda+1) + 3(3\lambda-2) - 2(-2\lambda-1) = 0$
$\Rightarrow \; 4\lambda + 2 + 9\lambda - 6 + 4\lambda + 2 = 0$
$\Rightarrow \; 17\lambda = 2$
$\lambda \;\; \lambda = 2/17$
$\therefore$ Direction ratio of required line $d_2 = (21, -28, -21)$
$= (3, -4, -3) = (-3, 4, 3)$
This line passes through $A(0, 1, 2)$.
$\therefore$ Required equation of line $\Rightarrow \frac{x-0}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Question159

Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$

is ..... .
[24 Feb 2021 Shift 2]

Answer: 1

Solution:

Solution:
Given, equation of line $\Rightarrow x-\lambda=2y-1=$

$\Rightarrow \quad \frac{x-\lambda}{1}=\frac{y-1/2}{\frac{1}{2}}=\frac{z}{-\frac{1}{2}}$

or $\quad \frac{x-\lambda}{2}=\frac{y-1/2}{1}=\frac{z}{-1}$

Point on this line through which it passes is $\left(\lambda, \frac{1}{2}, 0\right)$.

Equation of another line $\Rightarrow x=y+2\lambda=z-\lambda$

$\Rightarrow \quad \frac{x}{1}=\frac{y-(-2\lambda)}{1}=\frac{z-\lambda}{1}$ ... (ii)

A point through which this line passes is $(0, -2\lambda, \lambda)$.
Now, distance between two skew lines

$$=\frac{|(a_2-a_1)\cdot(b_1\times b_2)|}{|b_1\times b_2|}$$

According to the question, $\frac{\left|-5\lambda-\frac{3}{2}\right|}{\sqrt{14}}=\frac{\sqrt{7}}{2\sqrt{2}}$

$\Rightarrow \quad |10\lambda+3| \quad =7$

$10\lambda+3 \quad =\pm 7$

$\Rightarrow \quad 10\lambda \quad =4,-10$

$\Rightarrow \quad \lambda \quad =\frac{2}{5}$ and $\lambda=-1$

$\therefore \quad \lambda=-1$

$\left(\lambda=\frac{2}{5}\right.$ is not possible as λ is an integer)

$\therefore$

Hence, $\quad |\lambda|=|-1| \quad =1$

Question160

Let a, b ∈ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, −a − 9), then |a + b| is equal to
[24 Feb 2021 Shift 2]

Options:

A. 88

B. 86

C. 84

D. 90

Answer: A

Solution:

Solution:
Given, $P(a, 6, 9)$
Equation of line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$
Image of point P with respect to line is point $Q(20, b, -a-9)$
Mid-point of P and $Q = \left(\frac{a+20}{2}, \frac{6+b}{2}, \frac{-a}{2}\right)$
This point lies on line

$$\therefore \quad \frac{\frac{a+20}{2}-3}{7} = \frac{\frac{6+b}{2}-2}{5} = \frac{\frac{-a}{2}-1}{-9}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{a+2}{18} \text{ and } \frac{b+2}{10} = \frac{a+2}{18}$$

Solving, we get $a = -56, b = -32$
$\therefore \quad |a+b| = |-56-32| = 88$

Question161

Consider the three planes $P_1 : 3x + 15y + 21z = 9$, $P_2 : x - 3y - z = 5$ and $P_3 : 2x + 10y + 14z = 5$ Then, which one of the following is true? [26 Feb 2021 Shift 1]

Options:

A. P_1 and P_2 are parallel

B. P_1 and P_3 are parallel

C. P_2 and P_3 are parallel

D. P_1, P_2 and P_3 all are parallel

Answer: B

Solution:

Solution:
Given, $P_1 \Rightarrow 3x + 15y + 21z = 9$
$P_2 \Rightarrow x - 3y - z = 5$
$P_3 \Rightarrow 2x + 10y + 14z = 5$
Consider plane P_1, it can be written as
$3x + 15y + 21z = 9$ or $x + 5y + 7z = 3$
Again, consider plane P_3, it can be written as,
$2x + 10y + 14z = 5$ or $x + 5y + 7z = 5/2$
Hence, P_1 and P_3 are parallel.

Question162

If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals [26 Feb 2021 Shift 2]

Options:

A. 47

B. 43

C. 39

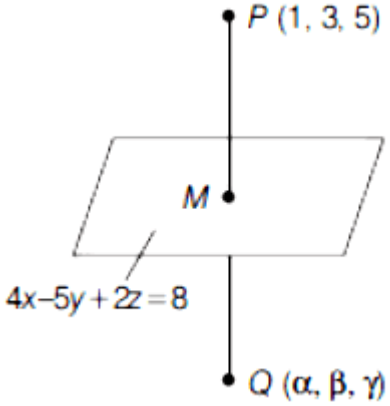
D. 41

Answer: A

Solution:

Solution:

Given, point $P(1, 3, 5)$ has the mirror image $Q(\alpha, \beta, \gamma)$ with respect to plane $4x - 5y + 2z = 8$



Then, M be the mid-point of line joining point P and Q. and M lies on given plane.
Coordinates of $M(a, b, c)$ are as follows

$a = \frac{\alpha+1}{2}, b = \frac{\beta+3}{2}, c = \frac{\gamma+5}{2}$

$\because$ M lies on plane, then

$4a - 5b + 2c = 8$

$4\left(\frac{\alpha+1}{2}\right) - 5\left(\frac{\beta+3}{2}\right) + 2\left(\frac{\gamma+5}{2}\right) = 8 \ldots$ (i)

Also, PQ is perpendicular to plane

$\Rightarrow \frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = \lambda$ (say)

$\Rightarrow \alpha = 4\lambda + 1, \beta = 3 - 5\lambda, \gamma = 2\lambda + 5 \ldots$ (ii)

Use Eq. (ii) in Eq. (i), we get

$2(4\lambda + 2) - 5\left(\frac{6-5\lambda}{2}\right) + 2\lambda + 10 = 8$

$\Rightarrow 8\lambda + 4 - 15 + \frac{25\lambda}{2} + 2\lambda + 10 = 8 \Rightarrow \lambda = \frac{2}{5}$

$\therefore \alpha = 4\lambda + 1 = 4\left(\frac{2}{5}\right) + 1 = \frac{13}{5}$

$\beta = 3 - 5\left(\frac{2}{5}\right) = \frac{5}{5} = 1$

and $\gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$

$\therefore 5(\alpha + \beta + \gamma) = 5\left(\frac{13}{5} + 1 + \frac{29}{5}\right) = 47$

Question163

Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals

[26 Feb 2021 Shift 2]

Options:

A. 142

B. 68

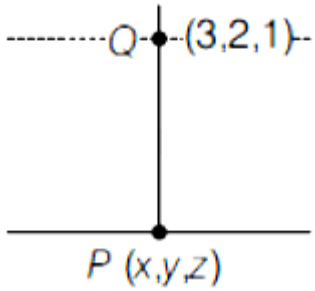
C. 136

D. 102

Answer: D

Solution:

Solution:
Given, $x+2y+z=6$ . . . (i)
and $y+2z=4$ . . . (ii)
Put $y=4-2z$ from Eq. (ii) Eq. in (i), we get
$x+8-4z+z=6$
$\Rightarrow x=-2+3z$
$\Rightarrow \frac{x+2}{3}=z$ . . . (iii)
$y=4-2z$
$\Rightarrow \frac{y-4}{-2}=z$ . . . (iv)
From Eqs. (iii) and (iv), line of intersection of two planes is
$\frac{x+2}{3}=\frac{y-4}{-2}=\frac{z}{1}=\lambda$
Then, Direction ratios of PQ is
$x-3\lambda-4-2\lambda, z=\lambda$
$(3\lambda-5,-2\lambda+2,\lambda-1)$
$(3\lambda-5,-2\lambda+2,\lambda-1)$



Since, PQ is perpendicular to the line, then
$3(3\lambda-5)-2(-2\lambda+2)+1(\lambda-1)=0$
$\therefore \lambda=\frac{10}{7}$
$\therefore P\left(\frac{16}{7},\frac{8}{7},\frac{10}{7}\right)$
Then, $21(\alpha+\beta+\gamma)=21\left(\frac{16}{7}+\frac{8}{7}+\frac{10}{7}\right)$
$=21\left(\frac{34}{7}\right)=3\times 34=102$

Question164

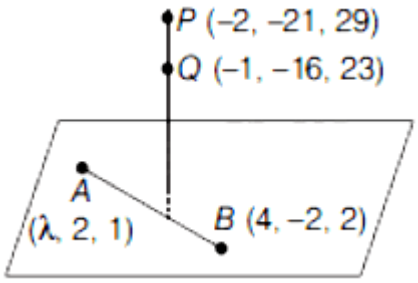
Let (λ, 2, 1) be a point on the plane which passes through the point (4, −2, 2). If the plane is perpendicular to the line joining the points (−2, −21, 29) and (−1, −16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is ..... .

[26 Feb 2021 Shift 1]

Answer: 8

Solution:

Solution:
Given $(\lambda, 2, 1)$ be point on the plane which passes through $(4, -2, 2)$ and plane is perpendicular to line joining P and Q.



Given, AB is perpendicular to PQ i.e., $AB \cdot PQ = 0$

Now, $AB = (4-\lambda)\hat{i} + (-2-2)\hat{j} + (2-\hat{k}$

$= (4-\lambda)\hat{i} - 4\hat{j} + \hat{k}$

$PQ = (-1+2)\hat{i} + (-16+21)\hat{j} + (23-29)\hat{k}$

$= \hat{i} + 5\hat{j} - 6\hat{k}$

$= (4-\lambda)\hat{i} - 4\hat{j} + \hat{k}$

$PQ = (-1+2)\hat{i} + (-16+21)\hat{j} + (23-29)\hat{k}$

$= \hat{i} + 5\hat{j} - 6\hat{k}$

Hence, $AB \cdot PQ = 0$

$\Rightarrow (4-\lambda)(1) + (-4)(5) + (1)(-6) = 0$

$\Rightarrow 4 - \lambda - 20 - 6 = 0$

$\Rightarrow \lambda = -22$

Then, $\left(\frac{\lambda}{11}\right)^2 - \left(\frac{4\lambda}{11}\right) - 4 = \left(\frac{-22}{11}\right)^2 - \left(\frac{4\times(-22)}{11}\right) - 4$

$= 4 - (-8) - 4 = 8$

Question165

A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If 0 is the origin and P is (2, −1, 1), then the projection of OP on this plane is of length
[2021 25 Feb Shift 2]

Options:

A. $\sqrt{\frac{2}{3}}$

B. $\sqrt{\frac{2}{11}}$

C. $\sqrt{\frac{2}{7}}$

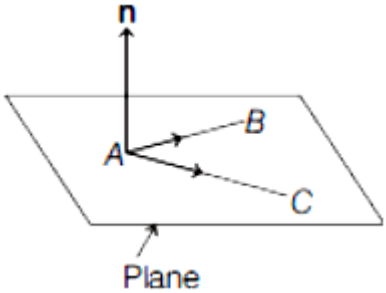
D. $\sqrt{\frac{2}{5}}$

Answer: B

Solution:

Solution:
Refer diagram, the normal vector be n and it is perpendicular to both AB and AC

$AB \times AC = n$



Now, $A(1, 2, 3), B(2, 3, 1)$ and $C(2, 4, 2)$

Then, $AB = (2-1)\hat{i} + (3-2)\hat{j} + (1-3)\hat{k}$

$= \hat{i} + \hat{j} - 2\hat{k}$

$-1)\hat{i} + (4-2)\hat{j} + (2-3)\hat{k}$

$AC = (2-1)\hat{i} + (4$

$= \hat{i} + 2\hat{j} - \hat{k}$

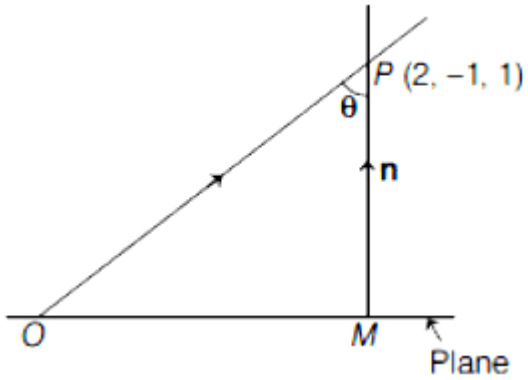
Now, $AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix}$

$= \hat{i}(-1+4) - \hat{j}(-1+2) + \hat{k}(2-1)$

$= 3\hat{i} - \hat{j} + \hat{k}$

$n = 3\hat{i} - \hat{j} + \hat{k}$

Let P be any point on normal vector and O be origin. Then refer the diagram, projection of OP on plane have length OM .



$OP = 2\hat{i} - \hat{j} + \hat{k}$ and $n = 3\hat{i} - \hat{j} + \hat{k}$

$OP.\ n = |OP|\,|n| \cos\theta$

$(6+1+1) = \sqrt{4+1+1}(\sqrt{9+1+1})\cos\theta$

$8 = \sqrt{6}\sqrt{11}\cos\theta \Rightarrow \cos\theta = \frac{8}{\sqrt{66}}$

Again, $\sin\theta = \frac{|OM|}{|OP|}$, gives $|OM| = \sin\theta \cdot |OP|$

$\Rightarrow |OM| = \sqrt{1-\cos^2\theta}\,|OP|$

$= \sqrt{1 - \frac{64}{66}}\sqrt{4+1+1} \quad \left(\text{use } \cos\theta = 8\sqrt{66}\right)$

$= \sqrt{\frac{2}{66}} \cdot \sqrt{6}$

$\therefore |OM| = \sqrt{\frac{2}{11}}$

Question166

The vector equation of the plane passing through the intersection of the planes $\mathbf{r} \cdot \left(\hat{i} + \mathbf{j} + \hat{k}\right) = 1$ and $\mathbf{r} \cdot \left(\hat{i} - 2\hat{j}\right) = -2$ and the point (1, 0, 2) is
[24 Feb 2021 Shift 2]

Options:

A. $\mathrm{r}\cdot\left(\hat{\mathrm{i}}+7\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=\frac{7}{3}$

B. $\mathrm{r}\cdot\left(3\hat{\mathrm{i}}+7\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=7$

C. $\mathrm{r}\cdot\left(\hat{\mathrm{i}}+7\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=7$

D. $\mathrm{r}\cdot\left(\hat{\mathrm{i}}-7\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=\frac{7}{3}$

Answer: C

Solution:

Solution:
Given, point $(1, 0, 2)$
Equation of plane =
$\mathrm{r}\cdot\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)=1$ and $\mathrm{r}\cdot\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}\right)=-2$
Equation of plane passing through the intersection of given planes is
$\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)-1\right]+\lambda\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}\right)+2\right]=0$
∵ This plane passes through point $(1, 0, 2)$ i.e.,
$\text{vector}\left(\hat{\mathrm{i}}+2\hat{\mathrm{k}}\right)$
$\therefore\ \left[\left(\hat{\mathrm{i}}+2\hat{\mathrm{k}}\right)\cdot\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)-1\right]+\lambda\left[\left(\hat{\mathrm{i}}+2\hat{\mathrm{k}}\right)\cdot\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}\right)+2\right]=0$
$\Rightarrow\ (3-1)+\lambda(1+2)=0$
$\Rightarrow 2+\lambda\times 3\ =0$
$\Rightarrow \lambda\ =-2/3$
Hence, equation of required plane is
$\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)-1\right]+\left(\frac{-2}{3}\right)\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}\right)+2\right]\ =0$

$\text{or}\quad 3\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)-1\right]-2\left[\mathrm{r}\cdot\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}\right)+2\right]\ =0$

$\text{or}\quad \mathrm{r}\cdot\left(\hat{\mathrm{i}}+7\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)\ =7$

Question167

The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane x + y + z = 17 is : 24 Feb 2021 Shift 1

Options:

A. $2\sqrt{19}$

B. $19\sqrt{2}$

C. 38

D. $\sqrt{38}$

Answer: D

Solution:

Solution:

Let $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = t$

$\Rightarrow x = 3 + t, y = 2t + 4, z = 2t + 5$

$3 + t + 2t + 4 + 2t + 5 = 17$

$\Rightarrow 5t = 5 \Rightarrow t = 1$

$\Rightarrow$ Point of intersection is $(4, 6, 7)$

Distance between $(1, 1, 9)$ and $(4, 6, 7)$ is

$\sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2}$

$= \sqrt{9 + 25 + 4} = \sqrt{38}$.

Question168

The equation of the plane passing through the point (1, 2, −3) and perpendicular to the planes 3x + y − 2z = 5 and 2x − 5y − z = 7, is 24 Feb 2021 Shift 1

Options:

A. $3x - 10y - 2z + 11 - 0$

B. $6x - 5y - 2z - 2 = 0$

C. $11x + y + 17z + 38 = 0$

D. $6x - 5y + 2z + 10 = 0$

Answer: C

Solution:

Solution:

Normal vector:

$$\begin{vmatrix} \hat{i} & j & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$$

So, direction ratios of normal to the required plane are $<11, 1, 17>$
Plane passes through $(1, 2, -3)$
So, equation of plane :
$11(x-1) + 1(y-2) + 17(z+3) = 0$
$\Rightarrow 11x + y + 17z + 38 = 0$

Question169

Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, −1, 2) and (−2, 1, −2), respectively. Let lines PR and QS intersect at T . If the vector TA is perpendicular to both PR and QS and the length of vector TA is $\sqrt{5}$ units, then the modulus of a position vector of A is
[16 Mar 2021 Shift 1]

Options:

A. $\sqrt{482}$

B. $\sqrt{171}$

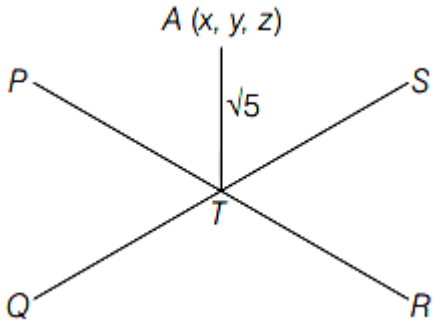
C. $\sqrt{5}$

D. $\sqrt{227}$

Answer: B

Solution:

Solution:



$P = (3\hat{i} - \hat{j} + 2\hat{k})$ and $Q = (\hat{i} + 2\hat{j} - 4\hat{k})$

$V_{PR} = (4, -1, 2)$ and $V_{QS}(-2, 1, -2)$

Equation of line $PR = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(4\hat{i} - \hat{j} + 2\hat{k})$

Equation of line $QS = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(-2\hat{i} + \hat{j} - 2\hat{k})$

Let T be the point of intersection.

$T = (3 + 4\lambda, -1 - \lambda, 2 + 2\lambda)$

$T = (1 - 2\mu, 2 + \mu, -4 - 2\mu)$

$3 + 4\lambda = 1 - 2\mu$

$\Rightarrow 2\lambda + \mu = -1$.....(i)

$-1 - \lambda = 2 + \mu$

$\Rightarrow \lambda + \mu = -3$.....(ii)

From Eqs. (i) and (ii),

$\lambda = 2$ and $\mu = -5$

$T = [3 + 4(2)], -1 - (2), 2 + 2(2) = (11, -3, 6)$

Now, DC of TA will be $V_{PR} \times V_{QS}$

$$\left| \hat{i}, \hat{j}, \hat{k}; -2, 1, -2; 4, -1, 2 \right| = 0\hat{i} - 4\hat{j} - 2\hat{k}$$

$L_{TA} \Rightarrow (11\hat{i} - 3\hat{j} + 6\hat{k}) + x(-4\hat{j} - 2\hat{k})$

Let $A = (11, -3 - 4x, 6 - 2x)$

$TA = \sqrt{5}$

$$\Rightarrow \sqrt{(11 - 11)^2 + (-3 - 4x + 3)^2 + (6 - 2x - 6)^2} = \sqrt{5}$$

$\Rightarrow (4x)^2 + (2x)^2 = 5 \Rightarrow 20x^2 = 5$

$\Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$

$A = [11, -3 - 4(1/2), 6 - 2(1/2)]$

$A = (11, -5, 5)$

Or

$A = [11, -3 + 4(1/2), 6 + 2(1/2)]$

$A = (11, -1, 7)$

$\therefore |A| = \sqrt{11^2 + 5^2 + 5^2}$ or

$\Rightarrow |A| = \sqrt{11^2 + 1^2 + 7^2}$

$\Rightarrow |A| = \sqrt{171}$ or $\sqrt{171}$

$\therefore |A| = \sqrt{171}$

Question170

If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}$, $I \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to [16 Mar 2021 Shift 2]

Options:

A. $1/2$

B. $1/\sqrt{6}$

C. $\sqrt{2/3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:

$L_1 \Rightarrow \frac{x-a}{I} = \frac{y-2}{3} = \frac{z-b}{4}$

Foot of perpendicular from $A(4, 3, 8)$ to L_1 is $B(3, 5, 7)$.

$AB = OB - OA$

$= \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + 8\hat{k}\right)$

$= -\hat{i} + 2\hat{j} - \hat{k}$

Now, AB is perpendicular to direction cosine of L_1,

So, $\left(-\hat{i} + 2\hat{j} - \hat{k}\right) \cdot \left(\hat{i} + 3\hat{j} + 4\hat{k}\right) = 0$

$\Rightarrow \quad -I + 6 - 4 = 0 \Rightarrow I = 2$

As, $(3, 5, 7)$ lies on L_1,

$\frac{3-a}{2} = \frac{5-2}{3} = \frac{7-b}{4}$

$3 - a = 2$

So, $a = 1, 7 - b = 4$

So, $b = 3$

$L_1 \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

and $L_2 = \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$\therefore$ Shortest distance will be along common normal.

So, common normal $= \begin{vmatrix} \hat{i}, \hat{j}, \hat{k}; 2, 3, 4; 3, 4, 5 \end{vmatrix}$

$\Rightarrow \quad n = -\hat{i} + 2\hat{j} - \hat{k} \Rightarrow \hat{n} = \frac{1}{\sqrt{6}}\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

Shortest distance will be the projection of $(2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}$ or $\hat{i}+2\hat{j}+2\hat{k}$ along $\hat{n}$

$\Rightarrow \left(\hat{i}+2\hat{j}+2\hat{k}\right)\dfrac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)}{\sqrt{6}}=\dfrac{-1+4-2}{\sqrt{6}}=\dfrac{1}{\sqrt{6}}$

Question171

Let P be an arbitrary point having sum of the squares of the distance from the planes $x+y+z=0$, $Ix-nz=0$ and $x-2y+z=0$, equal to 9 . If the locus of the point P is $x^2+y^2+z^2=9$, then the value of $I-n$ is equal to
[17 Mar 2021 Shift 2]

Answer: 0

Solution:

Solution:
Let $P=(\alpha, \beta, \gamma)$
Distance of point P from the plane $x+y+z=0$ is

$$=\frac{\alpha+\beta+\gamma}{\sqrt{3}}$$

Distance of point P from the plane $lx-nz=0$ is

$$=\frac{l\alpha-n\gamma}{\sqrt{l^2+n^2}}$$

and distance of point P from the plane $x-2y+z=0$ is

$$=\frac{\alpha-2\beta+\gamma}{\sqrt{6}}$$

According to the question,

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2+\left(\frac{lx-nz}{\sqrt{I^2+n^2}}\right)^2+\left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2=9$$

$\therefore$ Locus is

$$\frac{(x+y+z)^2}{3}+\frac{(lx-nz)^2}{I^2+n^2}+\frac{(x-2y+z)^2}{6}=9$$

$$\Rightarrow x^2\left(\frac{1}{2}+\frac{l^2}{l^2+n^2}\right)+y^2+z^2\left(\frac{1}{2}+\frac{n^2}{l^2+n^2}\right)+xz\left(1-\frac{2ln}{l^2+n^2}\right)=0$$

Comparing it with the given equation of locus, we get

$$2ln=I^2+n^2$$

$$\Rightarrow (I-n)^2=0$$

$$\Rightarrow I-n=0$$

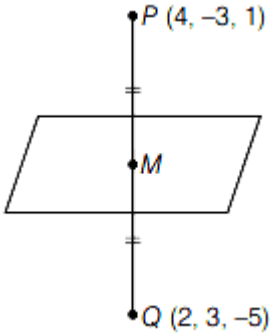
Question172

Let the plane, $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is .....
[18 Mar 2021 Shift 1]

Answer: 28

Solution:

Solution:
Let $P \equiv (4, -3, 1)$
and $Q \equiv (2, 3, -5)$



$\therefore \ M = \dfrac{P+Q}{2}$

$\Rightarrow \ M \equiv \left(\dfrac{4+2}{2}, \dfrac{-3+3}{2}, \dfrac{1-5}{2}\right)$

$\Rightarrow \ M \equiv (3, 0, -2)$

Also, direction ratios of $PQ = \{4-2, -3-3, 1+5\}$

$= \{2, -6, 6\}$

$\Rightarrow$ Direction ratios of $PQ = \{1, -3, 3\} =$ direction ratios of normal to the plane.

$\therefore$ Equation of the plane is

$1(x-3) - 3(y-0) + 3(z+2) = 0$

$\Rightarrow \ x - 3y + 3z + 3 = 0$

Comparing this to $ax + by + cz + d = 0$, we get

$a = 1, b - 3, c = 3, d = 3$

$\therefore$ Minimum value of $(a^2 + b^2 + c^2 + d^2) = 28$

Question173

The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point (1, 2, 3) is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is ..... .
[18 Mar 2021 Shift 1]

Answer: 4

Solution:

Solution:
Equation of any plane parallel to the plane $x - 2y + 2z - 3 = 0$ is
$x - 2y + 2z + \lambda = 0$....(i)
Given, distance from $(1, 2, 3)$ is 1.

$$\therefore \left| \frac{1 - 2 \times 2 + 2 \times 3 + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} \right| = 1$$

$\Rightarrow |\lambda + 3| = 3$
$\Rightarrow \lambda + 3 = \pm 3$
$\Rightarrow \lambda = 0, -6$
Consider, $\lambda = -6$
$\therefore$ Equation of required plane is
$x - 2y + 2z - 6 = 0$
On comparing this equation to
$ax + by + cz + d = 0$, we get
$a = 1, b = -2, c = 2$ and $d = -6$
$\because (b - d) = k(c - a)$
$\therefore 4 = k \times (1)$
$\Rightarrow k = 4$

Question174

The equation of the plane which contains the Y -axis and passes through the point (1, 2, 3) is
[17 Mar 2021 Shift 1]

Options:

A. $x + 3z = 10$

B. $x + 3z = 0$

C. $3x + z = 6$

D. $3x - z = 0$

Answer: D

Solution:

Solution:
Equation of plane passing through a point (x_1, y_1, z_1) is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
Here, $(x_1, y_1, z_1) = (1, 2, 3)$
So, $a(x - 1) + b(y - 2) + c(z - 3) = 0$
Now, Y -axis lies on it.
Direction ratio of Y -axis is $(0, 1, 0)$.
Normal vector to the plane $= a\hat{i} + b\hat{j} + c\hat{k}$
So, the normal vector of the plane will be perpendicular to direction ratio of Y -axis.
$a \cdot 0 + b \cdot 1 + c \cdot 0 = 0 \Rightarrow b = 0$
Equation of plane becomes
$a(x - 1) + c(z - 3) = 0$
Now, $x = 0, z = 0$ also satisfies the equation.
$a(0 - 1) + c(0 - 3) = 0$
$\Rightarrow \quad t \quad -a - 3c = 0 \Rightarrow a = -3c$
So, $-3c(x - 1) + c(z - 3) = 0$
$-3x + 3 + z - 3 = 0$
[as, $C \neq 0$]
$\Rightarrow \quad 3x - z = 0$

Question175

If for a > 0, the feet of perpendiculars from the points A(a, −2a, 3) and B(0, 4, 5) on the plane I x + my + nz = 0 are points C(0, −a, −1) and D respectively, then, the length of line segment CD is equal to [16 Mar 2021 Shift 1]

Options:

A. $\sqrt{31}$

B. $\sqrt{41}$

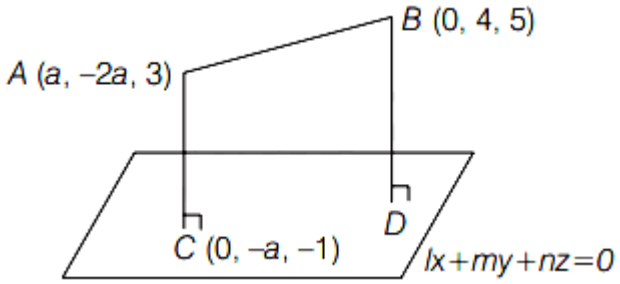
C. $\sqrt{55}$

D. $\sqrt{66}$

Answer: D

Solution:

Solution:



Given, $A \Rightarrow (a, -2a, 3)$
$B \Rightarrow (0, 4, 5)$
$C \Rightarrow (0, -a, -1)$
Equation of plane $P \Rightarrow I\,x + my + nz = 0$
As, C is foot of perpendicular from A to plane P. So, $CA \,|N$, where N is the normal vector to the plane.

$$CA = (a-0)\hat{i} + (-2a+a)\hat{j} + (3+1)\hat{k}$$
$$= a\hat{i} - a\hat{j} + 4\hat{k}$$

Now, $CA \,|N$

So, $\frac{a}{l} = \frac{-a}{m} = \frac{4}{n} = \lambda$

where λ is any real number.

$$P \Rightarrow \left(\frac{a}{\lambda}\right)x - \left(\frac{a}{\lambda}\right)y + \left(\frac{4}{\lambda}\right)z = 0$$

$$P \Rightarrow ax - ay + 4z = 0$$

C lies on plane.

So, $a \cdot 0 - a(-a) + 4(-1) = 0$

$$a^2 - 4 = 0 \Rightarrow a = \pm 2$$

As per the question, $a > 0$, so $a = 2$

So, equation of plane $P \Rightarrow 2x - 2y + 4z = 0$

$$P \Rightarrow x - y + 2z = 0$$

Coordinates of D

$$\frac{x-0}{1} = \frac{y-4}{-1} = \frac{z-5}{2} = \frac{-(0-4+10)}{[1^2+(-1)^2+2^2]}$$

If (x, y, z) be the foot of perpendicular drawn from (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$.

Then,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

Here, $(x, y, z) = (0, 4, 5)$

$$\Rightarrow \quad x - 0 = -(y-4) = \frac{z-5}{2} = \frac{-6}{6}$$

$$\therefore \quad x = -1, y = 5, z = 3$$

$$C = (0, -2, -1) \Rightarrow D = (-1, 5, 3)$$

$$\therefore \quad CD = \sqrt{(0+1)^2 + (-2-5)^2 + (-1-3)^2}$$

$$= \sqrt{1+49+16}$$

$$CD = \sqrt{66}$$

Question176

If (x, y, z) be an arbitrary point lying on a plane P, which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to

[16 Mar 2021 Shift 2]

Options:

A. 0

B. 3

C. 39

D. −45

Answer: B

Solution:

Solution:

Equation of plane passing through $A(42, 0, 0)$, $B(0, 42, 0)$ and $C(0, 0, 42)$ will be

$$\frac{x}{42}+\frac{y}{42}+\frac{z}{42}=1$$

$$\Rightarrow x+y+z=42$$

$$(x-11)+(y-19)+(z-12)=0$$

Now,

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}+\frac{y-19}{(x-11)^2(z-12)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{3(x-11)^2(y-19)^2(z-12)^2+(x-11)^3+(y-19)^3+(z-12)^3}{(x-11)^2(y-19)^2(z-12)^2}-\frac{42}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{(x-11)^3+(y-19)^3+(z-12)^3-3(x-11)(y-19)(z-12)+3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

$\Rightarrow$ If $A+B+C=0$

Then, $A^3+B^3+C^3=3ABC$

$$\Rightarrow (x-11)^3+(y-19)^3+(z-12)^3=3(x-11)(y-19)(z-12)$$

$$\Rightarrow \frac{3(x-11)(y-19)(z-12)-3(x-11)(y-19)(z-12)+3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

$$\Rightarrow \quad \frac{3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)} = 3$$

Question177

If the equation of the plane passing through the line of intersection of the planes
$2x - 7y + 4z - 3 = 0, 3x - 5y + 4z + 11 = 0$
and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is ..... .
[17 Mar 2021 Shift 1]

Answer: 4

Solution:

Solution:
Equation of the plane passing through the line of intersections of planes $2x - 7y + 4z - 3 = 0$ and $3x - 5y + 4z + 11 = 0$ is
$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$
Since this plane passes thought the point $(-2, 1, 3)$
$\therefore (-4 - 7 + 12 - 3) + \lambda(-6 - 5 + 12 + 11) = 0$
$-2 + 12\lambda = 0 \Rightarrow \lambda = 1/6$
$\therefore$ Equation of plane is
$(2x - 7y + 4z - 3) + \frac{1}{6}(3x - 5y + 4z + 11) = 0$
$15x - 47y + 28z - 7 = 0$
$\therefore a = 15, b = -47, c = 28$
$2a + b + c - 7 = 30 - 47 + 28 - 7 = 4$

Question178

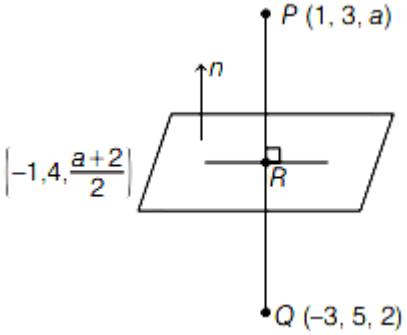
Let the mirror image of the point (1, 3, a) with respect to the plane
$\mathbf{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) - b = 0$ be $(-3, 5, 2)$. Then the value of $|a + b|$ is equal to
[18 Mar 2021 Shift 2]

Answer: 1

Solution:

Solution:

Given equation of plane in vector form is $\mathrm{r}\cdot\left(2\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)-\mathrm{b}=0$



Its Cartesian form will be

$2x-y+z=b$ ...(i)

$\because$R is the mid-point of PQ.

$\therefore\ \mathrm{R}\equiv\dfrac{\mathrm{P}+\mathrm{Q}}{2}\Rightarrow\mathrm{R}\equiv\left(-1,4,\ \dfrac{\mathrm{a}+2}{2}\right)$

$\because$R lies on the plane (i).

$\therefore\ -2-4+\dfrac{\mathrm{a}+2}{2}=\mathrm{b}\Rightarrow\mathrm{a}+2=2\mathrm{b}+12$

$\Rightarrow\ \mathrm{a}=2\mathrm{b}+10$...(ii)

$\because$ Direction ratio's of QP is $(1-(-3),3-5,\mathrm{a}-2)$

i.e. $(4,-2,\mathrm{a}-2)$

and direction ratios of normal to the given plane are $(2,-1,1)$

$\because$n and QP are parallel.

$\dfrac{2}{4}=\dfrac{-1}{-2}=\dfrac{1}{\mathrm{a}-2}$

$\therefore\ \mathrm{a}-2=2\Rightarrow\mathrm{a}=4$

From Eq. (ii), $\mathrm{b}=-3$

$\therefore\ \ |\mathrm{a}+\mathrm{b}|=|4-3|=|1|\ =1$

Question179

Let P be a plane containing the line $\dfrac{x-1}{3}=\dfrac{y+6}{4}=\dfrac{z+5}{2}$ and parallel to the line $\dfrac{x-3}{4}=\dfrac{y-2}{-3}=\dfrac{z+5}{7}$. If the point (1, −1, α) lies on the plane P, then the value of |5α| is equal to .....

[18 Mar 2021 Shift 2]

Answer: 38

Solution:

Solution:

Equation of required plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Since, $(1, -1, \infty)$ lies on it,
So, replace x by 1, y by (-1) and z and α

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$\Rightarrow \quad 5\alpha + 38 = 0 \Rightarrow 5\alpha = -38$
$\therefore |5\alpha| = |-38| = 38$

Question180

Let P be a plane l x + my + nz = 0 containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(−3, −6, 1) and B(2, 4, −3) in ratio k : 1, then the value of k is equal to
[16 Mar 2021 Shift 1]

Options:

A. 1.5

B. 3

C. 2

D. 4

Answer: C

Solution:

Solution:
$P \Rightarrow lx + my + nz = 0$
P contains L_1

$L_1 \Rightarrow \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$
So, $(1, -4, -2)$ lies on plane.
$1 - 4m - 2n = 0$...(i)
And $(-1, 2, 3)$ will be perpendicular to (l, m, n).
$-1 + 2m + 3n = 0$....(ii)
Adding Eqs. (i) and (ii),
$-2m + n = 0$
$n = 2m$
$l - 4m - 4m = 0$
$l = 8m$
So, $l = 8m$ and $n = 2m$
Plane $\Rightarrow 8x + y + 2z = 0$
Now, $A(-3, -6, 1)$ and $B(2, 4, -3)$
Plane P divides AB in the ratio of $k : 1$.
Let plane P intersect the line AB at point O.
So, $O = \left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$
And O lies on plane P,
So, $8(2k-3) + (4k-6) + 2(-3k+1) = 0$
$\Rightarrow 14k - 28 = 0$
$\therefore k = 2$

Question181

If the distance of the point (1, −2, 3) from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to.....

[16 Mar 2021 Shift 2]

Answer: 2

Solution:

Solution:
Given, point $A \Rightarrow (1, -2, 3)$
Plane $\Rightarrow x + 2y - 3z + 10 = 0$
Distance of point from plane along the vector $\left(3\hat{i} - m\hat{j} + \hat{k}\right)$ is $\sqrt{7/2}$.
Line passing through $(1, -2, 3)$ in the direction of
$\left(3\hat{i} - m\hat{j} + \hat{k}\right)$ is $\frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$

Any general point B will be $(3\lambda+1, -m\lambda-2, \lambda+3)$
Now, this point B lies on plane
So, $x+2y-3z+10=0$
$(3\lambda+1)+2(-m\lambda-2)-3(\lambda+3)+10=0$
$=(3-2m-3)\lambda=2$
$\Rightarrow \lambda=-1/m$
Now, $A=(1,-2,3)$
$B=(3\lambda+1, -m\lambda-2, \lambda+3)$
$|AB|^2=(3\lambda+1-1)^2+(-m\lambda-2+2)^2+(\lambda+3-3)^2$
$\Rightarrow 7/2=9\lambda^2+m^2\lambda^2+\lambda^2$
$\Rightarrow 7/2=10\lambda^2+1 \quad [\because m\lambda=-1]$
$\Rightarrow 10\lambda^2=5/2 \Rightarrow \lambda^2=1/4$
$\Rightarrow =\pm 1/2 \Rightarrow m=-1/\lambda$
$\therefore m=\pm 2$
and $|m|=2$

Question182

If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co-planar, then the value of k is _____.
[25 Jul 2021 Shift 2]

Answer: 1

Solution:

Solution:

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0$$

$(k+1)[2-6]-4[1-9]+6[2-6]=0$
$k=1$

Question183

If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3)$, $\lambda \in R$ is

$\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

[22 Jul 2021 Shift 2]

Options:

A. 3

B. 2

C. 5

D. -1

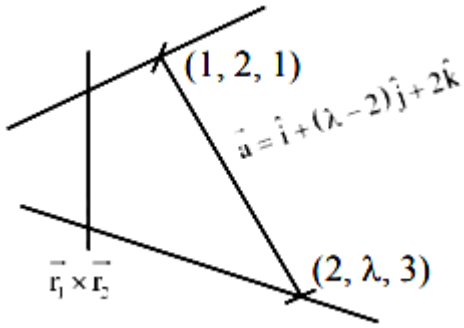
Answer: A

Solution:

Solution:

$L_1 : \frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-1)}{3}$ $\vec{r}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$

$L_2 : \frac{(x-2)}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4}$ $\vec{r}_2 = \hat{i} + 2\hat{j} + 4\hat{k}$



Shortest distance = Projection of $\vec{a}$ on $\vec{r}_1 \times \vec{r}_2$

$= \frac{\left|\vec{a} \cdot \left(\vec{r}_1 \times \vec{r}_2\right)\right|}{\left|\vec{r}_1 \times \vec{r}_2\right|}$

$\left|\vec{a} \cdot \left(\vec{r}_1 \times \vec{r}_2\right)\right| = \begin{vmatrix} 1 & \lambda-2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$

$\left|\vec{r}_1 \times \vec{r}_2\right| = \sqrt{38}$

$\therefore \frac{1}{\sqrt{38}} = \frac{|14-5\lambda|}{\sqrt{38}}$

$\Rightarrow |14 - 5\lambda| = 1$

$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \frac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$

Question184

If the shortest distance between the lines

$\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$, $\lambda \in R$, $\alpha > 0$and

overrightarrow mathrm $r_2 = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$, $\mu \in R$ is 9,

thenα is equal to _____.
[20 Jul 2021 Shift 1]

Answer: 6

Solution:

Solution:
If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$
then shortest distance between two lines is

$$L = \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|b \times d|}$$

$$\therefore \vec{a} - \vec{c} = \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\frac{\vec{b} \times \vec{d}}{|b \times d|} = \frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3}$$

$$\therefore \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot \frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3} = 9$$

or $\alpha = 6$

Question185

The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if :
[20 Jul 2021 Shift 2]

Options:

A. $b = 1, a \in R - \{0\}$

B. $a = 1, b \in R - \{0\}$

C. a = 2, b = 2

D. a = 2, b = 3

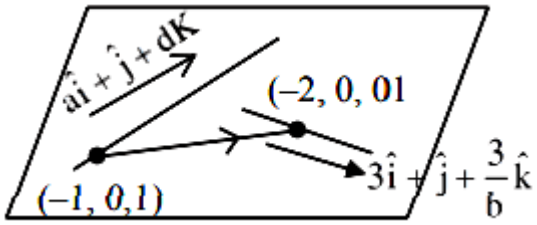
Answer: A

Solution:

Solution:

$\frac{x+1}{a} = y = \frac{z-1}{a}$

$\frac{x+2}{3} = y = \frac{z}{3/b}$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a-3) = 0$$

$a - \frac{3}{b} - a + 3 = 0$

$\sim b = 1, a \in R - \{0\}$

Question186

Let the plane passing through the point (−1, 0, −2) and perpendicular to each of the planes 2x + y − z = 2 and x − y − z = 3 be ax + by + cz + 8 = 0. Then the value of a + b + c is equal to: [27 Jul 2021 Shift 1]

Options:

A. 3

B. 8

C. 5

D. 4

Answer: D

Solution:

Solution:

Normal of req. plane $\left(2\hat{i}+\hat{j}-\hat{k}\right)\times\left(\hat{i}-\hat{j}-\hat{k}\right)$

$=-2\hat{i}+\hat{j}-3\hat{k}$

Equation of plane

$-2(x+1)+1(y-0)-3(z+2)=0$

$-2x+y-3z-8=0$

$2x-y+3z+8=0$

$a+b+c=4$

Question187

Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to: [25 Jul 2021 Shift 2]

Options:

A. $\frac{2}{\frac{1}{a}+\frac{1}{b}}$

B. $\frac{a+b}{2}$

C. $\frac{1}{a}+\frac{1}{b}$

D. $\sqrt{ab}$

Answer: D

Solution:

Solution:

Because vectors are coplanar

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0$

$\Rightarrow c^2=ab\Rightarrow c=\sqrt{ab}$

Question188

Let a plane P pass through the point (3, 7, −7) and contain the line, $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$. If distance of the plane P from the origin is d , then d^2 is equal to _____.
[27 Jul 2021 Shift 1]

Answer: 3

Solution:

Solution:
$\overrightarrow{BA}=\left(\hat{i}+4\hat{j}-5\hat{k}\right)$
$\overrightarrow{BA}=\left(\hat{i}+4\hat{j}-5\hat{k}\right)$

$$\overrightarrow{BA}\times\overrightarrow{l}=\overrightarrow{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 2 & 1\\ 1 & 4 & -5\end{vmatrix}$$

$a\hat{i}+b\hat{j}+c\hat{k}=-14\hat{i}-\hat{j}(14)+\hat{k}(-14)$
$a=1, b=1, c=1$
Plane is $(x-2)+(y-3)+(z+2)=0$
$x+y+z-3=0$
$d=\sqrt{3}\Rightarrow d^2=3$

Question189

For real numbers α and β ≠ 0, if the point of intersection of the straight lines $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$ lies on the plane x + 2y − z = 8, then α − β is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 5

B. 9

C. 3

D. 7

Answer: D

Solution:

Solution:
First line is $(\phi + \alpha, 2\phi + 1, 3\phi + 1)$
and second line is $(q\beta + 4, 3q + 6, 3q + 7)$.
For intersection
$\phi + \alpha = q\beta + 4$ .....(i)
$2\phi + 1 = 3q + 6$ .....(ii)
$3\phi + 1 = 3q + 7$.....(iii)
for (ii) & (iii) $\phi = 1, q = -1$
So, from (i) $\alpha + \beta = 3$
Now, point of intersection is $(\alpha + 1, 3, 4)$
It lies on the plane.
Hence, $\alpha = 5 \& \beta = -2$

Question190

The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, −4, −5) and R(2, −3, 1) and the plane 2x + y + z = 7, is equal to _____.
[27 Jul 2021 Shift 2]

Answer: 7

Solution:

Solution:
$\overrightarrow{QR} : -\frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$
$\Rightarrow (x, y, z) \equiv (r + 3, -r - 4, -6r - 5)$
Now, satisfying it in the given plane.
We get $r = -2$.
so, required point of intersection is $T(1, -2, 7)$.
Hence, $PT = 7$.

Question191

Let the foot of perpendicular from a point P(1, 2, −1) to the straight line L : $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N .
Let a line be drawn from P parallel to the plane x + y + 2z = 0 which meets L at point Q. If α is the acute angle between the lines PN and PQ, then cos α is equal to _____.
[25 Jul 2021 Shift 1]

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{\sqrt{3}}{2}$

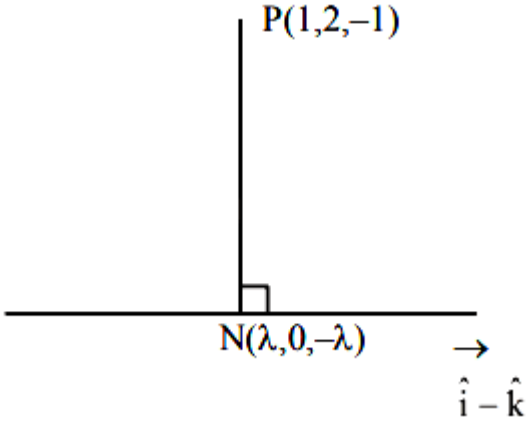
C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2\sqrt{3}}$

Answer: C

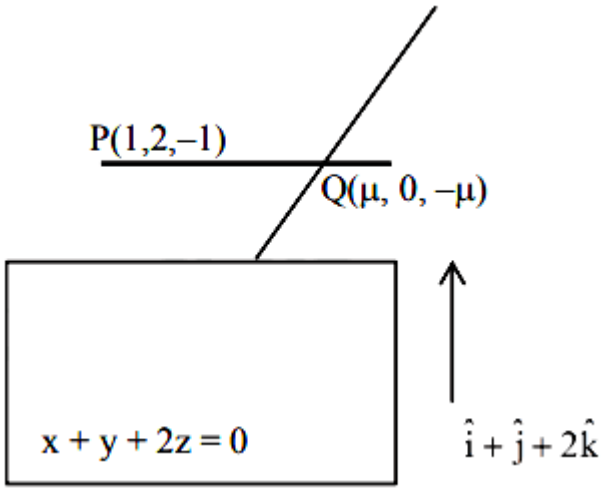
Solution:

Solution:



$\overrightarrow{PN} \cdot (\hat{i} - \hat{k}) = 0$
$\Rightarrow N(1, 0, -1)$

Now,



$\overrightarrow{PQ}.\left(\hat{i}+\hat{j}+2\hat{k}\right)=0$
$\Rightarrow \mu=-1$
$\Rightarrow Q(-1, 0, 1)$
$\overrightarrow{PN}=2\hat{j}$ and $\overrightarrow{PQ}=2\hat{i}+2\hat{j}-2\hat{k}$
$\Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

Question192

Let L be the line of intersection of planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=2$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=2$. If P(α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0), then thevalue of $35(\alpha+\beta+\gamma)$ is equal to :
[22 Jul 2021 Shift 2]

Options:

A. 101

B. 119

C. 143

D. 134

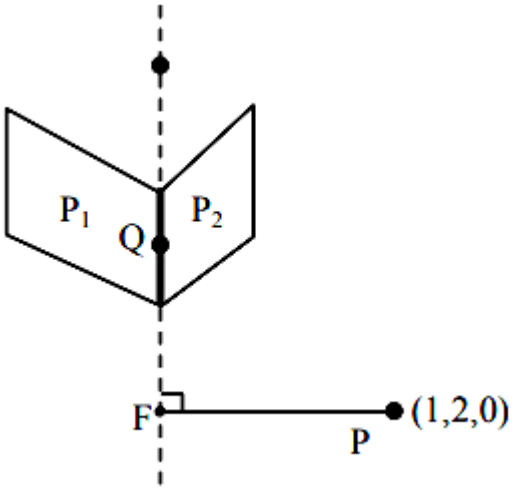
Answer: B

Solution:

Solution:

$P_1 : x - y + 2z = 2$

$P_2 = 2x + y - 3 = 2$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q.

$\Rightarrow$ z-coordinate of point Q is zero

$$\Rightarrow \left.\begin{matrix} x - y = 2 \\ \text{and } 2x + y = 2 \end{matrix}\right\} \Rightarrow x = \frac{4}{3}, y = \frac{-2}{3}$$

$$\Rightarrow Q\left(\frac{4}{3}, \frac{-2}{3}, 0\right)$$

Vector parallel to the line of intersection

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 5\hat{j} + 3\hat{k}$$

Equation of Line of intersection

$$\frac{x - \frac{4}{3}}{-1} = \frac{y + \frac{2}{3}}{5} = \frac{z - 0}{3} = \lambda(\text{ say })$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda + \frac{4}{3}, 5\lambda - \frac{2}{3}, 3\lambda\right)$$

$$\vec{PF} = \left(-\lambda + \frac{1}{3}\right)\hat{i} + \left(5\lambda - \frac{8}{3}\right)\hat{j} + (3\lambda)\hat{k}$$

$$\vec{PF} \cdot \vec{a} = 0$$

$$\Rightarrow \lambda - \frac{1}{3} + 25\lambda\frac{-40}{3} + 9\lambda = 0$$

$$\Rightarrow 35\lambda = \frac{41}{3} \Rightarrow \lambda = \frac{41}{105}$$

Now, $\alpha = -\lambda + \frac{4}{3}, \beta = 5\lambda - \frac{2}{3}, \gamma = 3\lambda$

$$\Rightarrow \alpha + \beta + \gamma = 7\lambda + \frac{2}{3}$$

$$= 7\left(\frac{41}{105}\right) + \frac{2}{3}$$

$$= \frac{51}{15}$$

$$\Rightarrow 35(\alpha + \beta + \gamma) = \frac{51}{15} \times 35 = 119$$

Question193

Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to theplane P, perpendicular to $\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{a} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____.
[20 Jul 2021 Shift 1]

Answer: 81

Solution:

Solution:
Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$
$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel$ to $3x - z - 2 = 0$
$\Rightarrow 3\alpha - 8 = 0$ .....(1)
$\vec{a} \perp \hat{i} + 2\hat{j} + 3\hat{k}$
$\Rightarrow \alpha + 2\beta + 38 = 0$.....(2)
$\vec{a} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 0$
$\Rightarrow \alpha + \beta + 28 = 2$ .....(3)
on solving 1, 2 & 3
$\alpha = 1, \beta = -5, 8 = 3$
So $(\alpha - \beta + 8) = 81$

Question194

Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, −1) with respect to L. Let a plane Pbe such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?
[20 Jul 2021 Shift 2]

Options:

A. $(-1, 1, 2)$

B. $(1, 1, 1)$

C. $(1, 1, 2)$

D. $(1, 2, 2)$

Answer: D

Solution:

Solution:
Plane $\mathrm{mathrm\,p}$ is $\perp^{r}$ to line
$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$
& passes through pt. $(2, 3)$ equation of plane p
$2(x-2)+1(y-3)+1(z+1)=0$
$2x+y+z-6=0$
pt $(1, 2, 2)$ satisfies above equation

Question195

The angle between the straight lines, whose direction cosines are given by the equations 2I + 2m − n = 0 and mn + nI + Im = 0, is [27 Aug 2021 Shift 2]

Options:

A. $\frac{\pi}{2}$

B. $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

C. $\cos^{-1}\left(\frac{8}{9}\right)$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Solution:
Given,
2l + 2m − n = 0 ....(i)
mn + nl + lm = 0 ...(ii)
From Eq. (i), we get
n = 2l + 2m ...(iii)
Substituting, n = 2l + 2m in Eq. (ii), we have
m(2l + 2m) + l(2l + 2m) + lm = 0

$\Rightarrow 2\,lm + 2m^2 + 2l^2 + 2\,lm + lm = 0$

$\Rightarrow 2l^2 + 4\,lm + lm + 2m^2 = 0$

⇒ 2l(l + 2m) + m(l + 2m) = 0
⇒ (2l + m) (l + 2m) = 0
When
2l = − m
From Eq (iii),
n = m

$\Rightarrow \frac{2l}{-1} = \frac{m}{1} = \frac{n}{1}$

$\Rightarrow \frac{l}{-\frac{1}{2}} = \frac{m}{1} = \frac{n}{1}$

or $\frac{l}{1} = \frac{m}{-2} = \frac{n}{-2}$

⇒ (l, m, n) = (1, − 2, − 2)
When
l = − 2m
From Eq. (iii),
n = − 2m
⇒ l = − 2m = n

$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-2}$

⇒ (l, m, n) = (− 2,1, − 2)
∴ Angles between straight lines

$$\cos\theta = \frac{\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)}{\left|\hat{i} - 2\hat{j} - 2\hat{k}\right| - 2\hat{i} + \hat{j} - 2\hat{k}\right|}$$

$\cos\theta = \frac{-2 - 2 + 4}{9} = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

Question196

The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane 2x − y + z = 6 from the point (−1, −1, 2) is

[31 Aug 2021 Shift 1]

Answer: 61

Solution:

Solution:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}=\lambda$$

$$\begin{cases} x=2\lambda+1 \\ y=3\lambda+2 \\ z=6\lambda-1 \end{cases}$$

Equation of plane is $2x-y+z=6$

$\Rightarrow 2(2\lambda+1)-(3\lambda+2)+(6\lambda-1)=6$

$7\lambda=7$

$\lambda=1$

P(3, 5, 5)

(Distance) $^2=(3+1)+(5+1)^2+(5-2)^2$

$=16+36+9=61$

Question197

The distance of the point (−1, 2, −2) from the line of intersection of the planes 2x + 3y + 2z = 0 and x − 2y + z = 0 is [31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{5}{2}$

C. $\frac{\sqrt{42}}{2}$

D. $\frac{\sqrt{34}}{2}$

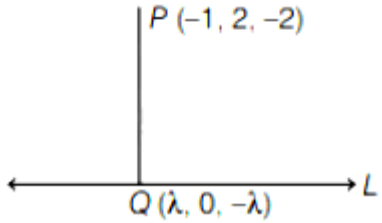
Answer: D

Solution:

Solution:
Let L be the line of intersection of $2x+3y+2z=0$ and $x-2y+z=0$
If $z=0$, then $x=y=0$
The line L is parallel to $r_1 \times r_2$, where $r_1 = 2\hat{i}+3\hat{j}+2\hat{k}$ and $r_2 = \hat{i}-2\hat{j}+\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 7\hat{i}-7\hat{k}$$

DR's of is $(1, 0, -1)$
and equation of L is $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}=\lambda$



Let PQ be the distance from the point $P(-1, 2, -2)$ to the line L.
DRs of $PQ = \lambda+1, -2, 2-\lambda$
$\therefore$ PQ $\perp$ r
$\Rightarrow(\lambda+1)(1)+(-2)(0)+(2-\lambda)(-1)=0$
$\Rightarrow\lambda+1-2+\lambda=0$
$\Rightarrow\lambda=\frac{1}{2}$

Coordinate of Q is $\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$$\Rightarrow PQ = \sqrt{\left(-1-\frac{1}{2}\right)^2+(2-0)^2+\left(-2+\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4}+4+\frac{9}{4}} = \frac{\sqrt{34}}{2}$$

Question198

If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is $\alpha x+\beta y+\gamma z=24$, then $\alpha+\beta+\gamma$ is equal to [17 Mar 2021 Shift 2]

Options:

A. 20

B. 19

C. 18

D. 21

Answer: B

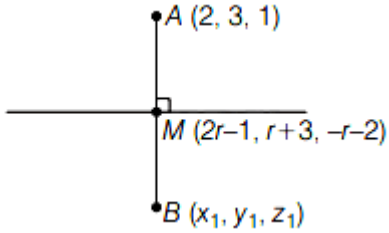
Solution:

Solution:
Let $A = (2, 3, 1)$

$L_1 \Rightarrow \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$

$L_2 \Rightarrow \frac{x-2}{3} = \frac{y-1}{-2} = \frac{z+1}{1}$

Any point M taken on L_1 is $(2r-1, r+3, -r-2)$



$\therefore$ Direction ratios of AM are $(2r-3, r, -r-3)$

$\because AM \perp L_1$

$\therefore 2(2r-3) + 1 \times r + (-1)(-r-3) = 0$

$\Rightarrow 4r - 6 + r + r + 3 = 0$

$\Rightarrow \ 6r = 3 \Rightarrow r = \frac{1}{2}$

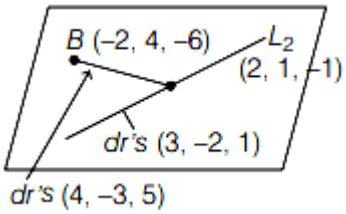
$\therefore \ M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore \ M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore \ B \equiv \left((2 \times 0) - 2, \left(2 \times \frac{7}{2}\right) - 3, \left(2 \times \left(\frac{-5}{2}\right)\right) - 1\right)$

$\Rightarrow \ B = (-2, 4, -6)$

Now, equation of plane containing $B(-2, 4, -6)$ and the line L_2 is



$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-9+8) = 0$

$\Rightarrow -7(x-2) - 11(y-1) - 1(z+1) = 0$

$\Rightarrow \ -7x - 11y - z = -14 - 11 + 1$

$\Rightarrow 7x + 11y + z = 24$ comparing this to

$\alpha x + \beta y + \gamma z = 24$

We get, $\alpha = 7, \beta = 11, \gamma = 1$

$\therefore \alpha + \beta + \gamma = 7 + 11 + 1 = 19$

Question199

Suppose the line $\frac{x-2}{\alpha}+\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x+3y-2z+\beta=0$. Then, $(\alpha+\beta)$ is equal to
[31 Aug 2021 Shift 2]

Answer: 7

Solution:

Solution:
Given equation of line
$\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ ...(i)
and plane $x+3y-2z+\beta=0$ ...(ii)
Line (i) pases through $(2, 2, -2)$
which lies on plane (ii).
$\therefore 2+6+4+\beta=0$
$\Rightarrow \beta = -12$
Also, given line is perpendicular to normal of the plane
$\alpha(1)-5(3)+2(-2)=0$
$\Rightarrow \alpha = 19$
$\therefore \alpha+\beta=7$

Question200

Let the equation of the plane, that passes through the point (1, 4, −3) and contains the line of intersection of the planes $3x-2y+4z-7=0$ and $x+5y-2z+9=0$ be $\alpha x+\beta y+\gamma z+3=0$, then $\alpha+\beta+\gamma$ isequal to
[31 Aug 2021 Shift 1]

Options:

A. -23

B. -15

C. 23

D. 15

Answer: A

Solution:

Solution:
Equation of plane is
$(3x-2y+4z-7)+\lambda(x+5y-2z+9)=0$
$(\lambda+3)x+(5\lambda-2)y+(4-2\lambda)z+9\lambda-7=0$
Passing through (1, 4, − 3)
$(\lambda+3)+4(5\lambda-2)-3(4-2\lambda)+9\lambda-7=0$
$\Rightarrow 36\lambda-24=0$
$\lambda=\frac{2}{3}$
⇒ Equation of plane
$\left(\frac{2}{3}+3\right)x+\left(\frac{10}{3}-2\right)y+\left(4-\frac{4}{3}\right)z+6-7=0$
$\Rightarrow 11x+4y+8z-3=0$
$\alpha=-11, \beta=-4, \gamma=-8$
$\alpha+\beta+\gamma=-23$

Question201

The distance of the point (1, −2, 3) from the plane x − y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, −6 is [27 Aug 2021 Shift 1]

Options:

A. 3

B. 5

C. 2

D. 1

Answer: D

Solution:

Solution:
Let A be any point on the plane $x-y+z=5$ and $B(1,-2,3)$.
Then equation of the line AB whose direction ratios are $2, 3, -6$
$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda$ (Let)
$\Rightarrow x=1+2\lambda, y=-2+3\lambda, z=3-6\lambda$
$A(1+2\lambda, -2+3\lambda, 3-6\lambda)$

A lies on plane.
Then, $1+2\lambda-(-2+3\lambda)+3-6\lambda=5$
$\Rightarrow 1+2\lambda+2-3\lambda+3-6\lambda=5$
$\Rightarrow \lambda=\frac{1}{7}$
$\therefore A\left(\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right)$
Distance AB $=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$
$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=1$

Question202

Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x-y-z-1=0$ and $2x+y-3z+4=0$, is
[27 Aug 2021 Shift 1]

Options:

A. $3x-y-5z+2=0$

B. $3x-4z+3=0$

C. $-x+2y+2z-3=0$

D. $4x-y-5z+2=0$

Answer: D

Solution:

Solution:
Given planes,
$x-y-z-1=0$ ...(i)
$2x+y-3z+4=0$ ...(ii)
Equation of plane passing through line of intersection of planes (i) and (ii) is given by
$(x-y-z-1)+\lambda(2x+y-3z+4)=0$
$\Rightarrow(2\lambda+1)x+(\lambda-1)y+(-3\lambda-1)z+(4\lambda-1)=0\ldots(iii)$
Distance of plane (iii) from origin $=\frac{2}{21}$(given)
$\Rightarrow\frac{|4\lambda-1|}{\sqrt{(2\lambda+1)^2+(\lambda-1)^2+(-3\lambda+1)^2}}=\sqrt{\frac{2}{21}}$

Squaring both sides

$$\frac{(4\lambda-1)^2}{(2\lambda+1)^2+(\lambda-1)^2+(3\lambda+1)^2}=\frac{2}{21}$$

$$\Rightarrow 21(16\lambda^2-8\lambda+1)=2(14\lambda^2+8\lambda+3)$$

$$\Rightarrow 308\lambda^2-184\lambda+15=0$$

$$308\lambda^2-154\lambda-30\lambda+15=0$$

$$(2\lambda-1)(154\lambda-15)=0$$

$$\Rightarrow \lambda=\frac{1}{2} \text{ or } \lambda=\frac{15}{154}$$

Putting $\lambda=\frac{1}{2}$ in Eq. (iii), we have

$$4x-y-5z+2=0$$

Question203

The equation of the plane passing through the line of intersection of the planes $\mathbf{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\mathbf{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the X-axis is
[27 Aug 2021 Shift 2]

Options:

A. $\mathbf{r}\cdot\left(\hat{j}-3\hat{k}\right)+6=0$

B. $\mathbf{r}\cdot\left(\hat{i}+3\hat{k}\right)+6=0$

C. $\mathbf{r}\cdot\left(\hat{i}-3\hat{k}\right)+6=0$

D. $\mathbf{r}\cdot\left(\hat{j}-3\hat{k}\right)-6=0$

Answer: A

Solution:

Solution:
Given, equation of planes

$$\mathbf{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1 \quad ...(i)$$

$$\mathbf{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0 \quad ...(ii)$$

Equation of plane passing through the intersection of the planes Eqs. (i) and (ii) is given by
$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$
or $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (-1 + 4\lambda) = 0$ ...(iii)
Plane (iii) in parallel to X-axis

1 + 2λ = 0 [Coefficient of x = 0]

$\Rightarrow \lambda = \frac{-1}{2}$

∴ From Eq. (iii) becomes

y − 3z + 6 = 0

or $\mathrm{r} \cdot \left(\hat{j} - 3\hat{k}\right) + 6 = 0$

Question204

Let S be the mirror image of the point Q (1, 3, 4) with respect to the plane$2x - y + z + 3 = 0$ and let R(3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is
[27 Aug 2021 Shift 2]

Answer: 72

Solution:

Solution:

Let point S(a, b, c)

Then,

$\frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = \frac{-2(2-3+4+3)}{4+1+1} = -2$

⇒ a = − 3, b = 5, c = 2

∴ S (− 3, 5, 2)

and point R(3, 5, γ) lies on the plane 2x − y + z + 3 = 0

⇒ 6 − 5 + γ + 3 = 0

⇒ γ = − 4

∴ R (3, 5, − 4)

Now, $SR^2 = 6^2 + 0^2 + (6)^2$

$= 36 + 0 + 36 = 72$

Question205

A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$ and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?
[26 Aug 2021 Shift 1]

Options:

A. $(-1, 1, 2)$

B. $(0, 1, 1)$

C. $(1, 0, 1)$

D. $(2, -1, 1)$

Answer: B

Solution:

Solution:
Equation of plane containing the given planes is
$(x + 2y + 3z + 1) + \lambda(x - y - z - 6) = 0$
$(1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + (1 - 6\lambda) = 0$
This plane is perpendicular to the plane $-2x + y + z + 8 = 0$
So, $-2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0$
$-2 - 2\lambda + 2 - \lambda + 3 - \lambda = 0$
$\lambda = \frac{3}{4}$
So, equation of required plane is $7x + 5y + 9z = 14$
Now, $(0, 1, 1)$ satisfies the above plane.

Question206

Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\mathbf{r} \cdot \left(\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\mathbf{r} \cdot \left(-\hat{i} + \hat{j} + \hat{k}\right) = 6$ Then which of the following points does not lie on P ?
[26 Aug 2021 Shift 2]

Options:

A. $(3, 3, 2)$

B. $(6, -6, 2)$

C. $(4, 2, 2)$

D. $(-8, 8, 6)$

Answer: C

Solution:

Solution:
P is a plane passing through the intersection of P_1 and P_2.
Equation of $P : P_1 + \lambda P_2 = 0$
$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$ ...(i)
Since plane P passes through $(1, 2, 3)$, then
$(1 + 2 + 12 - 16) + \lambda(-1 + 2 + 3 - 6) = 0$
$\Rightarrow -1 + \lambda(-2) = 0$
$\Rightarrow \lambda = \frac{-1}{2}$
On putting $\lambda = \frac{-1}{2}$ in Eq. (i), we get
$P : 3x + y + 7z - 26 = 0$
Clearly $(4, 2, 2)$ not lie on the plane.

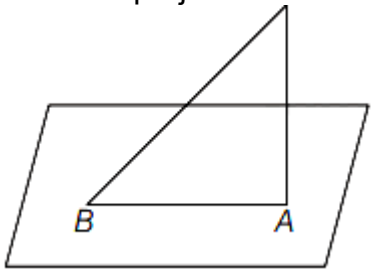
Question207

Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$.If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to
[26 Aug 2021 Shift 1]

Answer: 26

Solution:

Solution:
Given line, $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$
Given plane, $x - 2y - z = 3$
To find the projection let's find the foot of perpendicular from $(1, 3,, 4)$ to plane $x - 2y - z = 3$



$\frac{x-1}{1}=\frac{y-3}{-2}=\frac{z-4}{-1}=\lambda_1$

$(\lambda_1+1)-2(-2\lambda_1+3)-(-\lambda_1+4)=3$

$\Rightarrow 6\lambda_1=12$

$\Rightarrow \lambda_1=2$

So, foot of perpendicular from $(1, 3, 4)$ to plane $x-2y-z=3$ is A $(3, -1, 2)$.

Let us also find the intersection point of the plane and line

$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}=\lambda_2$

$(2\lambda_2+1)-2(\lambda_2+3)-(2\lambda_2+4)=3-2\lambda_2=12$

$\Rightarrow \lambda_2=-6$

The intersection point of the plane and line is $B(-11, -3, -8)$

Line passing through A and B is

$\frac{x-3}{-14}=\frac{y+1}{-2}=\frac{z-2}{-10}=\mu$

$\frac{x-3}{7}=\frac{y+1}{1}=\frac{z-2}{5}=\mu$

Now, let's find the distance from $O(0, 0, 6)$ to this line L.

Let's say $C(7\mu+3, \mu-1, 5\mu+2)$ is any point on L.

Then,

$\{(7\mu+3)-0\}.7+\{(\mu-1)-0\}.1+\{(5\mu+2)-6\}.5=0$

$\Rightarrow 49\mu+21+\mu-1+25\mu-20=0$

$\Rightarrow \mu = 0$

$C(3, -1, 2)$

Distance $=\sqrt{(3-0)^2+(-1-0)^2+(2-6)^2}=\sqrt{26}$

$d^2=26$

Question208

Let Q be the foot of the perpendicular from the point P(7, −2, 13) on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x+1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$.

Then $(PQ)^2$ is equal to
[26 Aug 2021 Shift 2]

Answer: 96

Solution:

Solution:

Plane containing the lines would be

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$\Rightarrow (x+1)(49-40)-(y-1)(42-24)+(z-3)(30-21)=0$
$\Rightarrow 9(x+1)-18(y-1)+9(z-3)=0$
$\Rightarrow x-2y+z=0$
Now, PQ will be equal to the perpendicular distance of the point $P(7,-2,13)$ from the plane $x-2y+z=0$

$$\therefore PQ = \left| \frac{7-2(-2)+13}{\sqrt{1^2+(-2)^2+(1)^2}} \right|$$

$$= \left| \frac{7+4+13}{\sqrt{1+4+1}} \right| = \left| \frac{24}{\sqrt{6}} \right| = 4\sqrt{6}$$

$PQ^2 = (4\sqrt{6})^2 = 16 \times 6 = 96$

Question209

The distance of line $3y-2z-1=0 = 3x-z+4$ from the point $(2,-1,6)$ is
[1 Sep 2021 Shift 2]

Options:

A. $\sqrt{26}$

B. $2\sqrt{5}$

C. C. $2\sqrt{6}$

D. $4\sqrt{2}$

Answer: C

Solution:

Solution:
Equation of line
$3y-2z-1=0=3x-z+4$

$$\Rightarrow \frac{3y-1}{2} = \frac{z-0}{1} = \frac{3x+4}{1}$$

$$\Rightarrow \frac{x+\frac{4}{3}}{1/3} = \frac{y-\frac{1}{3}}{2/3} = \frac{z-0}{1}$$

$$PR = |PQ|\cos\theta = |PQ|\frac{PQ \cdot P}{|PQ||P|} = \frac{PQ \cdot PQ}{|PR|}$$

$$PR = \left| \frac{\frac{1}{3}\left(2+\frac{4}{3}\right)+\frac{2}{3}\left(-1-\frac{1}{3}\right)+1(6-0)}{\sqrt{\frac{1}{9}+\frac{4}{9}+1}} \right| = 4\sqrt{\frac{14}{9}}$$

$$OR^2 = PQ^2 - PR^2$$

$$= \frac{100}{9}+\frac{16}{9}+36-\frac{224}{9}$$

$$= \frac{100}{9}+\frac{16}{9}+36-\frac{224}{9}$$

$$QR = \sqrt{24} = 2\sqrt{6}$$

Question210

Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then, which of the following points lies on P ?
[1 Sep 2021 Shift 2]

Options:

A. $\left(3, 1, -\frac{1}{2}\right)$

B. $\left(-2, 0, -\frac{1}{2}\right)$

C. $(0, 2, -4)$

D. $(4, 0, -2)$

Answer: B

Solution:

Solution:
Equation of angle bisectors

$$\frac{x-2y-2z+1}{\sqrt{1+4+4}} = \pm\frac{2x-3y-6z+1}{\sqrt{4+9+36}}$$

$$\Rightarrow x - 5y + 4z + 4 = 0 \text{ and } 13x - 23y - 32z + 10 = 0$$

Then, $\cos\theta = \frac{1+10-8}{\sqrt{1+4+4}\sqrt{1+25+16}} = \frac{1}{\sqrt{42}}$

$$\Rightarrow \tan\theta = \sqrt{41} > 1$$

$$\Rightarrow \theta > 45°$$

Then, acute angle bisector in plane

$P : 13x - 23y - 32z + 10 = 0$

$\therefore$ Point $\left(-2, 0, \frac{-1}{2}\right)$ lies on the plane P.

Question211

The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is:
[Jan. 08, 2020 (I)]

Options:

A. $2\sqrt{30}$

B. $\frac{7}{2}\sqrt{30}$

C. $3\sqrt{30}$

D. 3

Answer: C

Solution:

Solution:

$\overrightarrow{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$

$\overrightarrow{p} = \hat{i} + 4\hat{j} + 22\hat{k}$

$\overrightarrow{q} = \hat{i} + \hat{j} + 7\hat{k}$

$$\overrightarrow{p} \times \overrightarrow{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

Shortest distance between the lines is

$$= \frac{\left|\overrightarrow{AB} . \left(\overrightarrow{p} \times \overrightarrow{q}\right)\right|}{\left|\overrightarrow{p} \times \overrightarrow{q}\right|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}$$

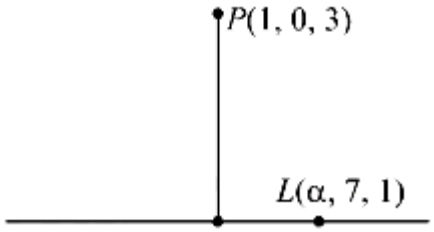
Question212

If the foot of the perpendicular drawn from the point (1,0,3) on a line passing through (α, 7, 1) is, then α is equal to _____. [NAJan.07, 2020 (II)]

Answer: 4

Solution:

Solution:
Since, PQ is perpendicular to L



$$\therefore \left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow \frac{2\alpha}{3}=\frac{24}{9} \Rightarrow \alpha=4$$

Question213

If for some α and β in R, the intersection of the following three planes
$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$
is a line in R^3, then $\alpha + \beta$ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. 0

B. 10

C. 2

D. –10

Answer: B

Solution:

Solution:

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$$\Rightarrow (7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0$$
$$\Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

Also, $D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$

$$\Rightarrow 1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0 \Rightarrow \beta = 13$$

Hence, $\alpha + \beta = -3 + 13 = 10$

Question214

If the distance between the plane, $23x - 10y - 2z + 48 = 0$and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}(\lambda \in R)$is equal to $\frac{k}{\sqrt{633}}$, then k is equal to _____.
[NA Jan. 9, 2020 (II)]

Answer: 3

Solution:

Solution:

Since, the line $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ contains the point (-1,3,-1) and line $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ contains the point(-3,-2,1)

Then, the distance between the plane$23x - 10y - 2z + 48 = 0$ and the plane containing the lines= perpendicular distance of plane $23x - 10y - 2z + 48 = 0$ either from (-1,3,-1) or (-3,-2,1)

$$= \left| \frac{23(-1) - 10(3) - 2(-1)}{\sqrt{(23)^2 + (10)^2 + (-2)^2}} \right| = \frac{3}{\sqrt{633}}$$

It is given that distance between the planes

$= \frac{k}{\sqrt{633}} \Rightarrow \frac{k}{\sqrt{633}} = \frac{3}{\sqrt{633}} \Rightarrow k = 3$

Question215

The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$.

Which of the following points lies on this plane?
[Jan. 8, 2020 (II)]

Options:

A. (1, 1, 1)

B. (1, –1, 1)

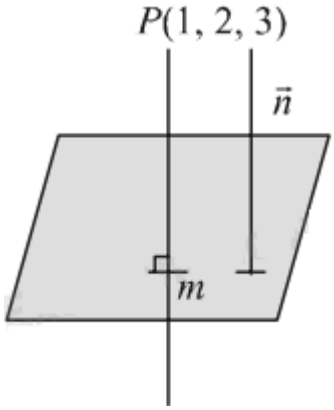
C. (–1, –1, 1)

D. (–1, –1, –1)

Answer: B

Solution:

Solution:



$\vec{n} = \frac{-7}{3} - 1, \frac{-4}{3} - 2, \frac{-1}{3} - 3$

$\vec{n} = \frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

D. r of normal to the plane (1,1,1)

Midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

∴ Equation of required plane Q

$\vec{r} \cdot \hat{n} = \hat{a} \cdot \hat{n}$

$\vec{r} \cdot \left(\hat{i}+\hat{j}+\hat{k}\right) = \frac{-2}{3}+\frac{1}{3}+\frac{4}{3}$

$\therefore$ Equation of plane is $x+y+z=1$

Question216

Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:
[Jan. 7, 2020 (I)]

Options:

A. (6, 5, 2)

B. (6, 5, –2)

C. (4, 3, 2)

D. (3, 4, –2)

Answer: B

Solution:

Solution:

Equation of plane is $x+y-2z=3$

$\Rightarrow \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$

$\Rightarrow (x, y, z) = (6, 5, -2)$

Question217

A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2) . Then the equation of the line through this centroid and perpendicular to the plane P is:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

B. $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

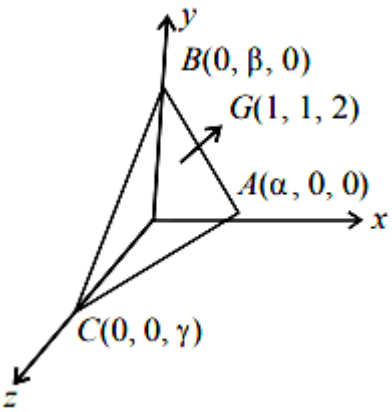
C. $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

D. $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

Answer: C

Solution:

Solution:



$\therefore \alpha = 3, \beta = 3$ and $\gamma = 6$ as G is centroid.
∴ The equation of plane is
$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$
$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1 \Rightarrow 2x+2y+z=6$
∴ The required line is, $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

Question218

If (a, b, c) is the image of the point (1,2,-3) in the line, $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$, then a + b + c is equals to:
[Sep. 05, 2020 (I)]

Options:

A. 2

B. – 1

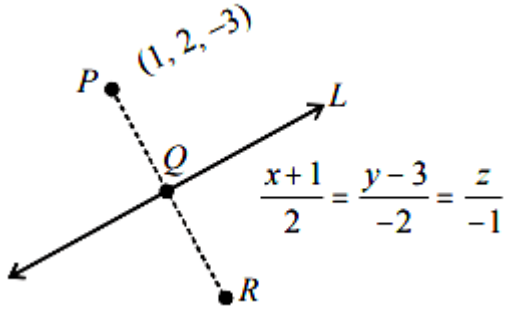
C. 3

D. 1

Answer: A

Solution:

Solution:
$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$
Any point on line $= Q(2\lambda - 1, -2\lambda + 3, -\lambda)$



$\therefore$ D.r. of PQ $= [2\lambda - 2, -2\lambda + 1, -\lambda + 3]$
D.r. of given line $= [2, -2, -1]$
$\because$PQ is perpendicular to line L
$\therefore 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$
$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$
$\Rightarrow 9\lambda - 9 = 0 \Rightarrow \lambda = 1$
$\because$Q is mid point of PR = Q = (1, 1, −1)
$\therefore$ Coordinate of image R = (1, 0, 1) = (a, b, c)
$\therefore a + b + c = 2$

Question219

The lines $\vec{r} = \left(\hat{i} - \hat{j}\right) + l\left(2\hat{i} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + m\left(\hat{i} + \hat{j} - \hat{k}\right)$ [Sep. 03, 2020 (I)]

Options:

A. do not intersect for any values of l and m

B. intersect for all values of l and m

C. intersect when l = 2 and m = $\frac{1}{2}$

D. intersect when l = 1 and m = 2

Answer: A

Solution:

Solution:

$L_1 \equiv \vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$

$L_2 \equiv \vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

Equating coeff. of $\hat{i}, \hat{j}$ and $\hat{k}$ of L_1 and L_2

$2l + 1 = m + 2$ .....(i)

$-1 = -1 + m \Rightarrow m = 0$ .....(ii)

$l = -m$ .....(iii)

$\Rightarrow m = l = 0$ which is not satisfy eqn. (i) hence lines do not intersect for any value of l and m.

Question220

The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0, 2x - y + z + 3 = 0$ is : [Sep. 06, 2020 (I)]

Options:

A. 1

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:

For line of intersection of planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$:

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Put $y = 0$, we get $x = -2$ and $z = 1$

$L_2 : \bar{r} = (-2\hat{i} + \hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and $L_1 : \bar{r} = (\hat{i} - \hat{j}) + \mu(-\hat{j} + \hat{k})$ (Given)

Now, $\bar{b}_1 \times \bar{b}_2 = -2\left[\hat{i}+\hat{j}+\hat{k}\right]$ and $\bar{a}_2 - oc\, a_1 = -3\hat{i}+\hat{j}+\hat{k}$

$\therefore$ Shortest distance $= \frac{1}{\sqrt{3}}$

Question221

If for some $\alpha \in R$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2passes through the point :
[Sep. 05, 2020 (II)]

Options:

A. (10,2,2)

B. (2,-10,-2)

C. (10,-2,-2)

D. (-2,10,2)

Answer: B

Solution:

Solution:

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-1-5+\alpha) - 3(2-\alpha) + 2(10-2\alpha+\alpha) = 0$

$\therefore \alpha = -4$

$\therefore$ Equation of $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

$\therefore$ Point (2,-10,-2) lies on line L_2

Question222

If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for

some a, b ∈ R, then the distance of the point (3,2,-1) from the plane P is _____.
[Sep. 04, 2020 (I)]

Answer: 3

Solution:

Solution:
Equation of plane P is
$(x+4y-z+7)+\lambda(3x+y+5z-8)=0$
$\Rightarrow x(1+3\lambda)+y(4+\lambda)+z(-1+5\lambda)+(7-8\lambda)=0$
$\Rightarrow \frac{1+3\lambda}{a}=\frac{4+\lambda}{b}=\frac{5\lambda-1}{6}=\frac{7-8\lambda}{-15}$
From last two ratios, $\lambda=-1$
$\Rightarrow \frac{-2}{a}=\frac{3}{b}=-1$
$\therefore a=2, b=-3$
∴ Equation of plane is, $2x-3y+6z-15=0$
Distance $=\frac{|6-6-6-15|}{7}=\frac{21}{7}=3.$

Question223

The distance of the point (1,-2,3) from the plane$x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is:
[NA Sep. 04, 2020 (II)]

Options:

A. $\frac{7}{5}$

B. 1

C. $\frac{1}{7}$

D. 7

Answer: B

Solution:

Solution:

Equation of line through point $P(1, -2, 3)$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

So, any point on line $= Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since, this point lies on plane $x - y + 2 = 5$

$\therefore 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$

$\therefore$ Point of intersection line and plane, $Q = \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance PQ

$= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = 1$

Question224

The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane: [Sep. 03, 2020 (I)]

Options:

A. $2x + y - z = 1$

B. $x - y - 2z = 1$

C. $x - 2y + z = 1$

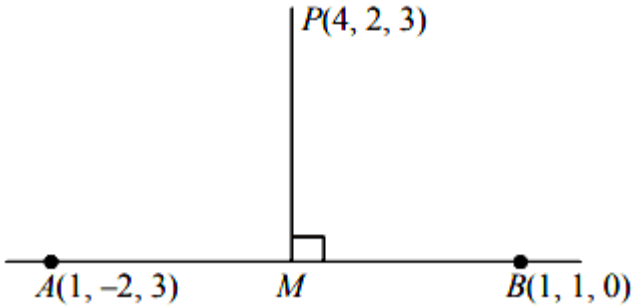
D. $x + 2y - z = 1$

Answer: A

Solution:

Solution:

Equation of line through points (1,-2,3) and (1,1,0) is



$\frac{x-1}{0}=\frac{y-1}{-3}=\frac{z-0}{3-0}(=\lambda$ say)

$\therefore M(1,-\lambda+1,\lambda)$

Direction ratios of PM $=[-3,-\lambda-1,\lambda-3]$

$\because PM \perp AB$

$\therefore(-3).0+(-1-\lambda)(-1)+(\lambda-3).1=0$

$\therefore\lambda=1$

$\therefore$ Foot of perpendicular $=(1,0,1)$

This point satisfy the plane $2x+y-z=1$

Question225

The plane which bisects the line joining the points (4, – 2, 3) and (2, 4, – 1) at right angles also passes through the point: [Sep. 03, 2020 (II)]

Options:

A. (4, 0, 1)

B. (0, –1, 1)

C. (4, 0, –1)

D. (0, 1, –1)

Answer: C

Solution:

Solution:

Direction ratios of normal to the plane are $<1,-3,2>$.

Plane passes through (3,1,1) .

Equation of plane is,

$1(x-3)-3(y-1)+2(z-1)=0$

$\Rightarrow x-3y+2z-2=0$

Question226

Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda\left(\hat{i} + \hat{j}\right)$, $\lambda \in R$ and $\vec{r} = -\hat{j} + \mu\left(\hat{j} - \hat{k}\right)$, $\mu \in R$ If Q(α, β, γ) is the foot of the perpendicular drawn from the point M (1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals _____.
[NA Sep. 03, 2020 (II)]

Answer: 5

Solution:

Solution:

Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$
Direction ratios of normal to the plane $= < -1, 1, 1 >$
Equation of plane
$-1(x-1) + 1(y-0) + 1(z-0) = 0$
$\Rightarrow x - y - z - 1 = 0$
If (x, y, z) is foot of perpendicular of $M(1, 0, 1)$ on the plane then
$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$
$\therefore x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$
$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$
$\therefore 3(\alpha + \beta + \gamma) = 3 \times \frac{5}{3} = 5$

Question227

The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also through the point :
[Sep. 02, 2020 (I)]

Options:

A. (0, 6, –2)

B. (–2, 0, 1)

C. (0, –6, 2)

D. (2, 0, –1)

Answer: B

Solution:

Solution:
Let plane passes through (2,1,2) be
$a(x-2)+b(y-1)+(z-2)=0$
It also passes through (1,2,1)
$\therefore -a+b-c=0 \Rightarrow a-b+c=0$
The given line is
$\frac{x}{3}=\frac{y}{2}=\frac{z-1}{0}$ is parallel to plane
$\therefore 3a+2b+c(0)=0$
$\Rightarrow \frac{a}{0-2}=\frac{b}{3-0}=\frac{c}{2+3}$
$\Rightarrow \frac{a}{2}=\frac{b}{-3}=\frac{c}{2+3}$
$\Rightarrow \frac{a}{2}=\frac{b}{-3}=\frac{c}{-5}$
$\therefore$ plane is $2x-4-3y+3-5z+10=0$
$\Rightarrow 2x-3y-5z+9=0$
The plane satisfies the point (-2,0,1) .

Question228

A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α , -3, 5), then α is equal to : [Sep. 02, 2020 (II)]

Options:

A. 5

B. –10

C. 10

D. -5

Answer: A

Solution:

Solution:
$\because$ Plane contains two lines

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$= -4\hat{i} + 5\hat{j} + 7\hat{k}$$

So, equation of plane is

$$-4(x-3) + 5(y-1) + 7(z-1) = 0$$

$$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$$

$$\Rightarrow -4x + 5y + 7z = 0$$

This also passes through $(\alpha, -3, 5)$

So, $-4\alpha - 15 + 35 = 0$

$$\Rightarrow -4\alpha = -20 \Rightarrow \alpha = 5$$

Question229

Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point- (-1,-1,1) . Then S is equal to: [Jan. 12, 2019 (II)]

Options:

A. $\{\sqrt{3}\}$

B. $\{\sqrt{3}, -\sqrt{3}\}$

C. $\{1, -1\}$

D. $\{3, -3\}$

Answer: B

Solution:

Solution:

Let $A(-\lambda^2, 1, 1), B(1, -\lambda^2, 1), C(1, 1, -\lambda^2), D(-1, -1, 1)$
lie on same plane, then

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$\Rightarrow(\lambda^2+1)((1-\lambda^2)^2-4)=0$
$\Rightarrow(3-\lambda^2)(\lambda^2+1)=0 \Rightarrow \lambda^2=3$
$\lambda=\pm\sqrt{3}$
Hence, $S=\{-\sqrt{3}, \sqrt{3}\}$

Question230

The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane $2x+3y-z=5$, contains which one of the following points?
[Jan. 11, 2019 (I)]

Options:

A. (2,2,0)

B. (-2,2,2)

C. (0,-2,2)

D. (2,0,-2)

Answer: D

Solution:

Solution:
Let normal to the required plane is $\vec{n}$
$\Rightarrow \vec{n}$ is perpendicular to both vector $2\hat{i}-\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}-3\hat{k}$

$$\Rightarrow \vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i}+8\hat{j}+8\hat{k}$$

$\Rightarrow$ Equation of the required plane is
$\Rightarrow(x-3)(-8)+(y+2)\times 8+(z-1)\times 8=0$
$\Rightarrow(x-3)(-1)+(y+2)\times 1+(z-1)\times 1=0$
$\Rightarrow x-3-y-2-z+1=0$

$\because x - y - z = 4$ passes through (2,0,-2)
$\therefore$ plane contains (2,0,-2)

Question231

The direction ratios of normal to the plane through the points (0,-1,0) and (0,0,1) and making an angle $\frac{\pi}{4}$with the plane $y - z + 5 = 0$ are:
[Jan. 11, 2019 (I)]

Options:

A. 2,-1,1

B. $2, \sqrt{2}, -\sqrt{2}$

C. $\sqrt{2}, 1, -1$

D. $2\sqrt{3}, 1, -1$

Answer: 0

Solution:

Solution:
(b,c) Let the d.r's of the normal be $\langle a, b, c \rangle$
Equation of the plane is
$a(x - 0) + b(y + 1) + c(z - 0) = 0$
$\because$ It passes through (0,0,1)
$\therefore b + c = 0$
Also $\frac{0 \cdot a + b - c}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{2}} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
$\Rightarrow b - c = \sqrt{a^2 + b^2 + c^2}$
And $b + c = 0$
$\Rightarrow b = \pm\frac{1}{\sqrt{2}}a$
$\therefore$ The d.r's are $\sqrt{2}, 1, -1$ or $2, \sqrt{2}, -\sqrt{2}$

Question232

If the point (2, α, β) lies on the plane which passes through the points (3,4,2) and (7,0,6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :
[Jan. 11, 2019(II)]

Options:

A. 12

B. 7

C. 5

D. 17

Answer: B

Solution:

Solution:
Let the normal to the required plane is $\vec{n}$, then

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i} + 8\hat{j} - 12\hat{k}$$

$\therefore$ Equation of the plane
$(x-3)\times 20 + (y-4)\times 8 + (z-2)\times(-12) = 0$
$5x - 15 + 2y - 8 - 3z + 6 = 0$
$5x + 2y - 3z - 17 = 0$ .....(1)
Since, equation of plane (1) passes through $(2, \alpha, \beta)$, then $10 + 2\alpha - 3\beta - 17 = 0 \Rightarrow 2\alpha - 3\beta = 7$

Question233

The plane which bisects the line segment joining the points (– 3, – 3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?
[Jan. 10, 2019 (II)]

Options:

A. (–2, 3, 5)

B. (4, – 1, 7)

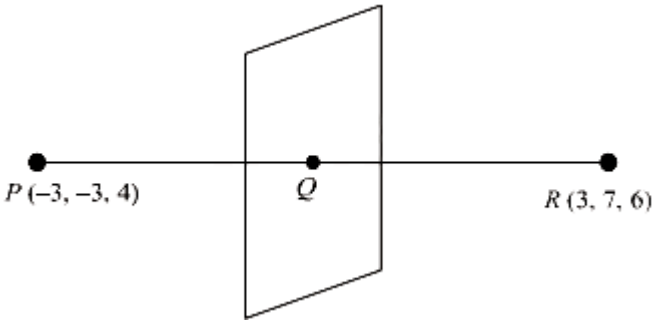
C. (2, 1, 3)

D. (4, 1, – 2)

Answer: D

Solution:

Solution:



Since, direction ratios of normal to the plane is
$\vec{n} = 6\hat{i} + 10\hat{j} + 2\hat{k}$
Then, equation of the plane is
$(x-0)6 + (y-2)10 + (z-5)2 = 0$
$3x + 5y - 10 + z - 5 = 0$
$3x + 5y + z = 15$
Since, plane (1) satisfies the point (4,1,-2)
Hence, required point is (4,1,-2)

Question234

On which of the following lines lies the point of in-ter-section of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane,$x + y + z = 2$? [Jan. 10, 2019 (II)]

Options:

A. $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

B. $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

C. $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

D. $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

Answer: C

Solution:

Solution:
Let any point on the line $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ be$A(2\lambda+4, 2\lambda+5, \lambda+3)$ which lies on the plane
$x+y+z=2$
$\Rightarrow 2\lambda+4+2\lambda+5+\lambda+3=2$
$\Rightarrow 5\lambda=-10 \Rightarrow \lambda=-2$
Then, the point of intersection is (0,1,1)
which lies on the line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

Question235

The system of linear equations
$x+y+z=2$
$2x+3y+2z=5$
$2x+3y+(a^2-1)z=a+1$
[Jan 09 2019I]

Options:

A. is inconsistent when a = 4

B. has a unique solution for |a| = $\sqrt{3}$

C. has infinitely many solutions for a = 4

D. is inconsistent when |a| = $\sqrt{3}$

Answer: D

Solution:

Solution:
Since the system of linear equations are
$x+y+z=2$ .....(1)
$2x+3y+2z=5$ .....(2)
$2x+3y+(a^2-1)z=a+1$ .....(1)

Now, $\Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{vmatrix}$
(Applying $R_3 \rightarrow R_3-R_2$)

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$$

$= a^2 - 3$

When, $\Delta = 0 \Rightarrow a^2 - 3 = 0 \Rightarrow |a| = \sqrt{3}$

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are parallel.
Hence, the given system of equation is inconsistent.

Question236

The equation of the line passing through (−4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is: [Jan 09 2019I]

Options:

A. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

B. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

C. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

D. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

Answer: C

Solution:

Solution:
Let any point on the intersecting line

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda$ (say)
is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$
Since, the above point lies on a line which passes through the point (-4,3,1)
Then, direction ratio of the required line
$= < -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 >$
or $< -3\lambda + 3, 2\lambda, -\lambda + 1 >$
Since, line is parallel to the plane
$x + 2y - z - 5 = 0$
Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$
Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$

$\Rightarrow \lambda = -1$
Now direction ratio of the required line $= < 6, -2, 2 >$ or $<3, -1, 1>$
Hence required equation of the line is
$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

Question237

The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y -axis also passes through the point:
[Jan 09 2019I]

Options:

A. (-3,0,-1)

B. (-3,1,1)

C. (3,3,-1)

D. (3,2,1)

Answer: D

Solution:

Solution:
Since, equation of plane through intersection of planes
$x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is
$(2x + 3y - z + 4) + \lambda (x + y + z - 1) = 0$
$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ .....(1)
But, the above plane is parallel to y -axis then
$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$
$\Rightarrow \lambda = -3$
Hence, the equation of required plane is
$-x - 4z + 7 = 0$
$\Rightarrow x + 4z - 7 = 0$
Therefore, (3,2,1) the passes through the point.

Question238

The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:
[Jan. 09, 2019 (II)]

Options:

A. $x - 2y + z = 0$

B. $3x + 2y - 3z = 0$

C. $x + 2y - 2z = 0$

D. $5x + 2y - 4z = 0$

Answer: A

Solution:

Solution:
Let the direction ratios of the plane containing lines
$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $<a, b, c>$
$\therefore 3a + 4b + 2c = 0$
$4a + 2b + 3c = 0$
$\therefore \frac{a}{12 - 4} = \frac{b}{8 - 9} = \frac{c}{6 - 16}$
$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$
$\therefore$ Direction ratio of plane $= < -8, 1, 10>$
Let the direction ratio of required plane is $<l, m, n>$
Then $-8l + m + 10n = 0$ .....(1)
and $2l + 3m + 4n = 0$ ..... (2)
From (1) and (2),
$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$
$\therefore$ D.R.s are $<1, -2, 1>$
$\therefore$ Equation of plane: $x - 2y + z = 0$

Question239

A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(−1, 1, 2)and O(0, 0, 0). The angle between the faces OPQ and PQR is:
[Jan.12, 2019 (I)]

Options:

A. $\cos^{-1}\left(\frac{17}{31}\right)$

B. $\cos^{-1}\left(\frac{19}{35}\right)$

C. $\cos^{-1}\left(\frac{9}{35}\right)$

D. $\cos^{-1}\left(\frac{7}{31}\right)$

Answer: B

Solution:

Solution:
Let $\vec{v}_1$ and $\vec{v}_2$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{v}_1 = \overline{PQ} \times \overline{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

$$\vec{v}_2 = \overline{PQ} \times \overline{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\because \cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{5+5+9}{25+1+9} = \frac{19}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

Question240

Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$intersect at the point R. The reflection of R in the xy - plane has coordinates : [Jan. 11, 2019 (II)]

Options:

A. (2, –4, –7)

B. (2, 4, 7)

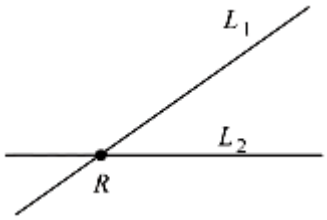
C. (2, –4, 7)

D. (–2, 4, 7)

Answer: A

Solution:

Solution:
Let the coordinate of P with respect to line



$\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1} = \lambda$

$\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4} = \mu$

$L_1 = (\lambda + 3, 3\lambda - 1, -\lambda + 6)$

and coordinate of P w.r.t.
line $L_2 = (7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\therefore \lambda - 7\mu = -8, 3\lambda + 6\mu = 3, \lambda + 4\mu = 3$

From above equation : $\lambda = -1, \mu = 1$

$\therefore$ Coordinate of point of intersection $R = (2, -4, 7)$

Image of R w.r.t. xy plane $= (2, -4, -7)$

Question241

If the lines x = ay + b, z = cy + d and x = a' z + b', y = c' z + d' are perpendicular, then : [Jan. 09, 2019 (II)]

Options:

A. ab' + bc' + 1 = 0

B. cc' + a + a' = 0

C. bb' + cc' + 1 = 0

D. $aa' + c + c' = 0$

Answer: D

Solution:

Solution:
First line is: $x = ay + b, z = cy + d$
$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$
and another line is: $x = a'z + b', y = c'z + d'$
$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$
$\because$ Both lines are perpendicular to each other
$\therefore aa' + c' + c = 0$

Question242

The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and $\frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$, is : [Jan. 12, 2019 (I)]

Options:

A. $11\sqrt{6}$

B. $11/\sqrt{6}$

C. 11

D. $6\sqrt{11}$

Answer: B

Solution:

Solution:
$\because$ plane containing both lines.

D.R. of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$

Now, equation of plane is,
$7(x-1) - 14(y-4) + 7(z+4) = 0$
$\Rightarrow x - 1 - 2y + 8 + z + 4 = 0$

$\Rightarrow x-2y+z+11=0$
Hence, distance from (0,0,0) to the plane,
$=\frac{11}{\sqrt{1+4+1}}=\frac{11}{\sqrt{6}}$

Question243

If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kx=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is

[Jan. 12, 2019 (II)]

Options:

A. $\sqrt{\frac{5}{3}}$

B. $\sqrt{\frac{3}{5}}$

C. $-\frac{3}{5}$

D. $-\frac{5}{3}$

Answer: A

Solution:

Solution:
Let angle between line and plane is θ, then
$\sin\theta=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$
$=\left|\frac{\left(2\hat{i}+\hat{j}-2\hat{k}\right).\left(\hat{i}-2\hat{j}-K\hat{k}\right)}{\sqrt{9}.\sqrt{1+4+K^2}}\right|$
$=\left|\frac{2-2_2K}{3\sqrt{5+K^2}}\right|=\frac{2|K|}{3\sqrt{4}+K^2}$
Since, $\cos\theta=\frac{2\sqrt{2}}{3}\Rightarrow\sin\theta=\frac{1}{3}$

Then, $\frac{2|K|}{3\sqrt{5}+K^2}=\frac{1}{3}\Rightarrow 4K^2=5+K^2$

$3K^2=5\Rightarrow K=\pm\sqrt{\frac{5}{3}}$

Question244

If the length of the perpendicular from the point (β, 0, β)($\beta\neq 0$) to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to:

[April 10, 2019 (I)]

Options:

A. 1

B. 2

C. –1

D. –2

Answer: C

Solution:

Solution:

Given, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=p$(let) and point $P(\beta, 0, \beta)$

Any point on line $A=(p, 1, -p-1)$

Now, DR of AP a" $<p-\beta, 1-0, -p-1-\beta>$

Which is perpendicular to line.

$\therefore(p-\beta)1+0.1-1(-p-1-\beta)=0$

$\Rightarrow p-\beta+p+1+\beta=0\Rightarrow p=\frac{-1}{2}$

$\therefore$ Point $A\left(\frac{-1}{2}, 1-\frac{1}{2}\right)$

Given that distance $AP=\sqrt{\frac{3}{2}}\Rightarrow AP^2=\frac{3}{2}$

$\Rightarrow\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2}$ or $2\left(\beta+\frac{1}{2}\right)^2=\frac{1}{2}$

$\Rightarrow\left(\beta+\frac{1}{2}\right)^2=\frac{1}{4}\Rightarrow\beta=0, -1, (\beta\neq 0)$

$\therefore\beta=-1$

Question245

The vertices B and C of a "ABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, −1, 2), is:
[April 09, 2019 (II)]

Options:

A. $5\sqrt{17}$

B. $2\sqrt{34}$

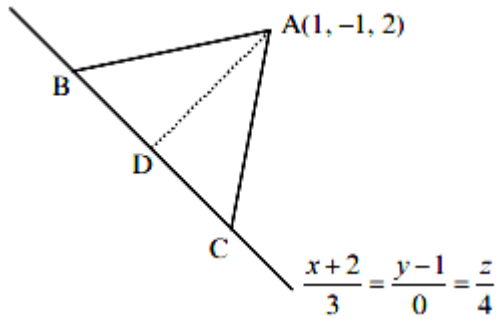
C. 6

D. $\sqrt{34}$

Answer: D

Solution:

Solution:
Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$
$\overline{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$
$\because \overline{AD} \perp \overline{BC}$,
$\therefore \overline{AD} \cdot \overline{BC} = 0$
$\Rightarrow (3\lambda - 3) + 3 + 2(0) + (4\lambda - 2)4 = 0$
$\Rightarrow \lambda = \frac{17}{25}$



Hence, D = $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$|\overline{AD}| = \sqrt{\left(\frac{1}{25} - 1\right)^2 + (2)^2 + \left(\frac{68}{25} - 2\right)^2}$

$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25}} = \sqrt{\frac{3400}{25}} = \frac{2\sqrt{34}}{5}$

Area of triangle $= \frac{1}{2} \times \left|\overline{BC}\right| \times \left|\overline{AD}\right|$

$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

Question246

If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then distance of R from the origin is : [April 08, 2019 (II)]

Options:

A. $2\sqrt{14}$

B. $2\sqrt{21}$

C. 6

D. $\sqrt{53}$

Answer: A

Solution:

Solution:
Here, P, Q, R are collinear

$\therefore \overline{PR} = \lambda \overline{PQ}$

$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda\left[6\hat{i} + 3\hat{j} + 6\hat{k}\right]$

$\Rightarrow 6\lambda = 2,\ y+3 = 3\lambda,\ z-4 = 6\lambda$

$\Rightarrow \lambda = \frac{1}{3},\ y = -2,\ z = 6$

$\therefore$ Point $R(4, -2, 6)$

Now, $OR = \sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{56} = 2\sqrt{14}$

Question247

A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are : [April 10, 2019 (II)]

Options:

A. (1, 0, 2)

B. (2, 0, 1)

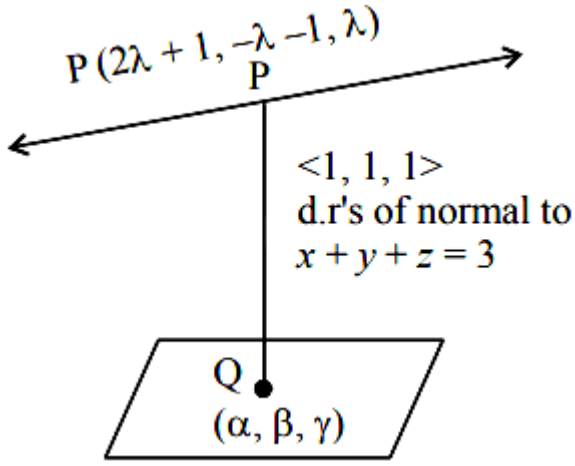
C. (–1, 0, 4)

D. (4, 0, –1)

Answer: B

Solution:

Solution:



Let co-ordinates of Q be (α, β, γ), then
$\alpha+\beta+\gamma=3$ .....(i)
$\alpha-\beta+\gamma=3$ .....(ii)
$\Rightarrow \alpha+\gamma=3$ and $\beta=0$
Equating direction ratio's of PQ, we get
$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$
$\Rightarrow \alpha=3\lambda+2, \gamma=2\lambda+1$
Substituting the values of α and γ in equation (i), we get
$\Rightarrow 5\lambda+3=3 \Rightarrow \lambda=0$
Hence, point is Q(2, 0, 1)

Question248

The length of the perpendicular from the point (2,-1,4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :
[April 08, 2019 (I)]

Options:

A. greater than 3 but less than 4

B. less than 2

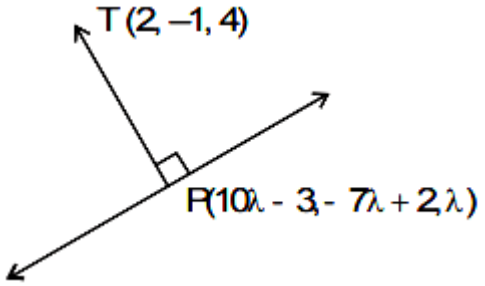
C. greater than 2 but less than 3

D. greater than 4

Answer: A

Solution:

Solution:
Let P be the foot of perpendicular from point $T\,(2, -1, 4)$ on the given line. So P can be assumed as $P\,(10\lambda - 3, -7\lambda + 2, \lambda)$



DR's of T P propto to $10\lambda - 5, -7\lambda + 3, \lambda - 4$
$\because$T P and given line are perpendicular, so $10(10\lambda - 5) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$
$\Rightarrow \lambda = \frac{1}{2}$
$\Rightarrow T\,P = \sqrt{(10\lambda - 5)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$
$= \sqrt{0 + \frac{1}{4} + \frac{49}{4}} = \sqrt{12.5} = 3.54$
Hence, the length of perpendicular is greater than 3 but less than 4

Question249

If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane$2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to:
[April 12, 2019 (I)]

Options:

A. 14

B. $\sqrt{14}$

C. $2\sqrt{7}$

D. $2\sqrt{14}$

Answer: D

Solution:

Solution:
Let points $P(3\lambda+2, 2\lambda-1, -\lambda+1)$ and $Q(3\mu+2, 2\mu-1, -\mu+1)$
$\because P$ lies on $2x+3y-z+13=0$
$\therefore 6\lambda+4+6\lambda-3+\lambda-1+13=0$
$\Rightarrow 13\lambda=-13 \Rightarrow \lambda=-1$
Hence, $P(-1,-3,2)$
Similarly, Q lies on $3x+y+4z=16$
$\therefore 9\mu+6+2\mu-1-4\mu+4=16$
$\Rightarrow 7\mu=7 \Rightarrow \mu=1$
Hence, Q is (5,1,0)
Now, $PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

Question250

A plane which bisects the angle between the two given planes 2x – y + 2z – 4 = 0 and x + 2y + 2z – 2 = 0, passes through the point : [April 12, 2019 (II)]

Options:

A. (1, –4, 1)

B. (1, 4, –1)

C. (2, 4, 1)

D. (2, –4, 1)

Answer: D

Solution:

Solution:
The equations of angle bisectors are,
$x+2y+2z-23=\pm\frac{2x-y+2z-4}{3}$
$\Rightarrow x-3y-2=0$

or $3x + y + 4z - 6 = 0$
(2,-4,1) lies on the second plane.

Question251

The length of the perpendicular drawn from the point (2, 1,4) to the plane containing the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ is :
[April 12, 2019 (II)]

Options:

A. 3

B. $\frac{1}{3}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: C

Solution:

Solution:

The equation of plane containing two given lines is, $\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$

On expanding, we get $x - y - z = 0$
Now, the length of perpendicular from (2,1,4) to this plane
$= \left| \frac{2-1-4}{\sqrt{1^2+1^2+1^2}} \right| = \sqrt{3}$

Question252

If Q(0, −1, −3) is the image of the point P in the plane 3x− y + 4z = 2 and R is the point (3, −1, −2), then the area (in sq. units) of Δ PQR is

:

[April 10, 2019 (I)]

Options:

A. $2\sqrt{13}$

B. $\frac{\sqrt{91}}{4}$

C. $\frac{\sqrt{91}}{2}$

D. $\frac{\sqrt{65}}{2}$

Answer: C

Solution:

Solution:
Image of $Q(0, -1, -3)$ in plane is,

$$\frac{(x-0)}{3} = \frac{(y+1)}{-1} = \frac{z+3}{+4} = \frac{-2(1-12-2)}{9+1+16} = 1$$

$\Rightarrow x = 3, y = -2, z = 1$

$\Rightarrow P(3, -2, 1), Q(0, -1, -3), R(3, -1, -2)$

$\therefore$ Area of ΔPQR is

$$\frac{1}{2}\left|\overrightarrow{QP} \times \overrightarrow{QR}\right| = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|\left\{\hat{i}(-1) - \hat{j}(3-12) + \hat{k}(3)\right\}\right|$$

$$= \frac{1}{2}\sqrt{(1+81+9)} = \frac{\sqrt{91}}{2}$$

Question253

If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{2}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to:

[April 10,2019 (II)]

Options:

A. 9

B. 15

C. 5

D. 13

Answer: D

Solution:

Solution:
Let, $P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given, distance between P_1 and P_2 is $\frac{1}{3}$

$$\frac{1}{3} = \frac{\left|3 - \frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3 - \frac{\lambda}{2}\right| = 1 \Rightarrow \lambda_{max} = 8$$

And distance between P_1 and P_3 is $\frac{2}{3}$

$$\frac{2}{3} = \frac{|\mu - 3|}{\sqrt{9}} \Rightarrow \mu_{max} = 5$$

$$\Rightarrow (\lambda + \mu)_{max} = 13$$

Question254

If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane,$x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is:
[April 09 2019 I]

Options:

A. $\sqrt{5}/2$

B. $2\sqrt{5}$

C. $9/2$

D. $7/2$

Answer: C

Solution:

Solution:
Let point on line be $P(2k+1, 3k-1, 4k+2)$
Since, point P lies on the plane $x+2y+3z=15$
$\therefore 2k+1+6k-2+12k+6=15$
$\Rightarrow k=\frac{1}{2}$
$\therefore P \equiv \left(2, \frac{1}{2}, 4\right)$
Then the distance of the point P from the origin is $OP = \sqrt{4+\frac{1}{4}+16} = \frac{9}{2}$

Question255

A plane passing through the points (0,-1,0) and (0,0,1)and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point: [April 09 2019 I]

Options:

A. $(-\sqrt{2}, 1, -4)$

B. $(\sqrt{2}, -1, 4)$

C. $(-\sqrt{2}, -1, -4)$

D. $(\sqrt{2}, 1, 4)$

Answer: D

Solution:

Solution:
Let the required plane passing through the points(0,-1,0) and (0,0,1) be $\frac{x}{\lambda}+\frac{y}{-1}+\frac{z}{1}=1$ and the given plane is $y-z+5=0$
$\therefore \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$
$\Rightarrow \lambda^2 = \frac{1}{2} \Rightarrow \frac{1}{\lambda} = \pm\sqrt{2}$

Then, the equation of plane is $\pm\sqrt{2}x - y + z = 1$
Then the point $(\sqrt{2}, 1, 4)$ satisfies the equation of plane

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Question256

Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy -plane. Then the distance of the point (0,0,256) from P is equal to:
[April 09, 2019 (II)]

Options:

A. $17/\sqrt{5}$

B. $63\sqrt{5}$

C. $205\sqrt{5}$

D. $11/\sqrt{5}$

Answer: D

Solution:

Solution:
Let the plane be
$P \equiv (2x + 3y + z + 5) + \lambda(x + y + z - 6) = 0$
$\because$ above plane is perpendicular to xy plane.
$\therefore \left((2+\lambda)\hat{i} + (3+\lambda)\hat{j} + (1+\lambda)\hat{k}\right) . \hat{k} = 0 \Rightarrow \lambda = -1$
Hence, the equation of the plane is,
$P \equiv x + 2y + 11 = 0$
Distance of the plane P from (0,0,256)
$\left|\frac{0+0+11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$

Question257

The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point

(1,1,0) is:
[April 08 2019 I]

Options:

A. $x-3y-2z=-2$

B. $2x-z=2$

C. $x-y-z=0$

D. $x+3y+z=4$

Answer: C

Solution:

Solution:
Let the equation of required plane be;
$(2x-y-4)+\lambda(y+2z-4)=0$
$\because$ This plane passes through the point (1,1,0) then $(2-1-4)+\lambda(1+0-4)=0$
$\Rightarrow\lambda=-1$
Then, equation of required plane is,
$(2x-y-4)-(y+2z-4)=0$
$\Rightarrow 2x-2y-2z=0 \Rightarrow x-y-z=0$

Question258

The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5which is perpendicular to the plane x − y + z = 0 is:
[April 08,2019 (II)]

Options:

A. $\vec{r}\times\left(\hat{i}-\hat{k}\right)+2=0$

B. $\vec{r}\,.\left(\hat{i}-\hat{k}\right)-2=0$

C. $\vec{r}\times\left(\hat{i}+\hat{k}\right)+2=0$

D. $\vec{r}\,.\left(\hat{i}-\hat{k}\right)+2=0$

Answer: D

Solution:

Solution:
Equation of the plane passing through the line of intersection of $x+y+z=1$ and $2x+3y+4z=5$ is
$(2x+3y+4z-5)+\lambda(x+y+z-1)=0$
$\Rightarrow(2+\lambda)x+(3+\lambda)y+(4+\lambda)z+(-5-\lambda)=0$ .....(i)
$\because$ plane(i) is perpendicular to the plane $x-y+z=0$
$\therefore(2+\lambda)(1)+(3+\lambda)(-1)+(4+\lambda)(1)=0$
$2+\lambda-3-\lambda+4+\lambda=0 \Rightarrow \lambda=-3$
Hence, equation of required plane is
$-x+z-2=0$ or $x-z+2=0$
$\Rightarrow \bar{r}.\left(\hat{i}-\hat{k}\right)+2=0$

Question259

The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is:
[2018]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

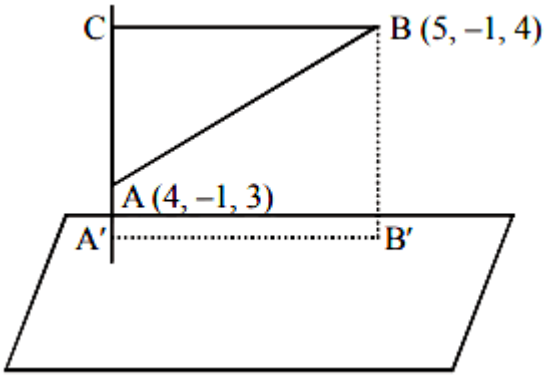
C. $\sqrt{\frac{2}{3}}$

D. $\frac{2}{\sqrt{3}}$

Answer: C

Solution:

Solution:



$AC = \overrightarrow{AB} \cdot \hat{AC} = (\hat{i} + \hat{k}) \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2 - \frac{4}{3}} = \sqrt{\frac{2}{3}}$

$\therefore$ Length of projection $= \sqrt{\frac{2}{3}}$

Question260

An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is [Online April 15, 2018]

Options:

A. $\cos^{-1}\left(\frac{1}{8}\right)$

B. $\cos^{-1}\left(\frac{1}{6}\right)$

C. $\cos^{-1}\left(\frac{1}{3}\right)$

D. $\cos^{-1}\left(\frac{1}{4}\right)$

Answer: B

Solution:

Solution:
Given
$l + 3m + 5n = 0$ .....(1)

and $5lm - 2mn + 6nl = 0$ .....(2)
From eq. (1) we have
$l = -3m - 5n$
Put the value of l in eq. (2), we get;
Put the value of l in eq. (2), we get;
$5(-3m - 5n)m - 2mn + 6n(-3m - 5n) = 0$
$\Rightarrow 15m^2 + 45mn + 30n^2 = 0$
$\Rightarrow m^2 + 3mn + 2n^2 = 0$
$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$
$\Rightarrow (m + n)(m + 2n) = 0$
$\therefore m = -n$ or $m = -2n$
For $m = -n, l = -2n$
And for $m = -2n, l = n$
$\therefore (l, m, n) = (-2n, -n, n)$ Or $(l, m, n) = (n, -2n, n)$
$\Rightarrow (l, m, n) = (-2, -1, 1)$ Or $(l, m, n) = (1, -2, 1)$
Therefore, angle between the lines is given as:

$$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}.\sqrt{6}}$$

$$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

Question261

If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{P} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then P is equal to

[Online April 16, 2018]

Options:

A. $-\frac{7}{4}$

B. $\frac{2}{7}$

C. $-\frac{4}{7}$

D. $\frac{7}{2}$

Answer: D

Solution:

Solution:

Let θ be the angle between the two lines

Here direction cosines of $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ are 2,2,1

Also second line can be written as:

$$\frac{x-5}{2}=\frac{y-2}{\frac{P}{7}}=\frac{z-3}{4}$$

$\therefore$ its direction cosines are $2, \frac{P}{7}, 4$

Also, $\cos\theta=\frac{2}{3}$ (Given)

$$\because \cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$\Rightarrow\frac{2}{3}=\left|\frac{(2\times2)+\left(2\times\frac{P}{7}\right)+(1\times4)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+\frac{P^2}{49}+4^2}}\right|$$

$$=\frac{4+\frac{2P}{7}+4}{3\times\sqrt{2^2+\frac{P^2}{49}+4^2}}$$

$$\Rightarrow\left(4+\frac{P}{7}\right)^2=20+\frac{P^2}{49}\Rightarrow16+\frac{8P}{7}+\frac{P^2}{49}=20+\frac{P^2}{49}$$

$$\Rightarrow\frac{8P}{7}=4\Rightarrow P=\frac{7}{2}$$

Question262

If L_1 is the line of intersection of the planes $2x-2y+3z-2=0, x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0, 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is: [2018]

Options:

A. $\frac{1}{3\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{4\sqrt{2}}$

Answer: A

Solution:

Solution:
Equation of plane passing through the line of intersection of first two planes is:

$(2x-2y+3z-2)+\lambda(x-y+z+1)=0$
or $x(\lambda+2)-y(2+\lambda)+z(\lambda+3)+(\lambda-2)=0$ .....(i)
is having infinite number of solution with
$x+2y-z-3=0$ and $3x-y+2z-1=0$, then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

Now put $\lambda=5$ in (i), we get
$7x-7y+8z+3=0$

Now perpendicular distance from (0,0,0) to the place containing L_1 and $L_2 = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

Question263

The sum of the intercepts on the coordinate axes of the plane passing through the point (-2,-2,2) and containing the line joining the points (1,-1,2) and (1,1,1) is
[Online April 16, 2018]

Options:

A. 12

B. -8

C. -4

D. 4

Answer: C

Solution:

Solution:

Equation of plane passing through three given points is:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 1+2 & -1+2 & 2-2 \\ 1+2 & 1+2 & 1-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -x+3y+6z-8=0$$

$$\Rightarrow \frac{x}{8}-\frac{3y}{8}-\frac{6z}{8}+\frac{8}{8}=0$$

$$\Rightarrow \frac{x}{8}-\frac{y}{\frac{8}{3}}-\frac{z}{\frac{8}{6}}=-1$$

$$\Rightarrow \frac{x}{-8}+\frac{y}{\frac{8}{3}}+\frac{z}{\frac{8}{6}}=1$$

$\therefore$ Sum of intercepts $= -8+\frac{8}{3}+\frac{8}{6}=-4$

Question264

A variable plane passes through a fixed point (3,2,1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz− plane through A, a second plane is drawn parallel zx− plane through B and a third plane is drawn parallel to xy− plane through C. Then the locus of the point of intersection of these three planes, is [Online April 15, 2018]

Options:

A. $x+y+z=6$

B. $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

C. $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

D. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

Answer: C

Solution:

Solution:
If a, b, c are the intercepts of the variable plane on the x, y, z axes respectively, then the equation of the plane is
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
And the point of intersection of the planes parallel to the xy, yz and zx planes is (a, b, c).
As the point (3,2,1) lies on the variable plane, so
$\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$
Therefore, the required locus is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

Question265

An angle between the plane, x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z − 1 = 0 and 5x + 8y + 2z + 14 = 0, is [Online April 15, 2018]

Options:

A. $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$

B. $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

C. $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$

D. $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Answer: D

Solution:

Solution:

Normal to $3x+4y+z=1$ is $3\hat{i}+4\hat{j}+\hat{k}$.

Normal to $5x+8y+2z=-14$ is $5\hat{i}+8\hat{j}+2\hat{k}$

The line of intersection of the planes is perpendicular to both normals, so, direction ratios of the intersection line are directly proportional to the cross product of the normal vectors.

Therefore the direction ratios of the line is $-\hat{j}+4\hat{k}$

Hence the angle between the plane $x+y+z+5=0$ and the intersection line is

$$\sin^{-1}\left(\frac{-1+4}{\sqrt{17}\sqrt{3}}\right)=\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$$

Question266

A plane bisects the line segment joining the points (1, 2, 3) and (– 3, 4, 5) at right angles. Then this plane also passes through the point. [Online April 15, 2018]

Options:

A. (– 3, 2, 1)

B. (3, 2, 1)

C. (1, 2, – 3)

D. (– 1, 2, 3)

Answer: A

Solution:

Solution:

Since the plane bisects the line joining the points (1,2,3) and (-3,4,5) then the plane passes through the midpoint of the line which is:

$$\left(\frac{1-3}{2},\frac{2+4}{2},\frac{5+3}{2}\right)\equiv\left(\frac{-2}{2},\frac{6}{2},\frac{8}{2}\right)\equiv(-1,3,4)$$

As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are $(-3-1,4-2,5-3)=(-4,2,2)$

So the equation of the plane is :$-4x+2y+2z=\lambda$

As plane passes through (-1,3,4) so

$-4(-1)+2(3)+2(4)=\lambda\Rightarrow\lambda=18$

Therefore, equation of plane is :$-4x+2y+2z=18$

Now, only (-3,2,1) satisfies the given plane as

$-4(-3)+2(2)+2(1)=18$

Question267

If the image of the point P(1, −2, 3) in the plane,$2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$is Q, then PQ is equal to : [2017]

Options:

A. $6\sqrt{5}$

B. $3\sqrt{5}$

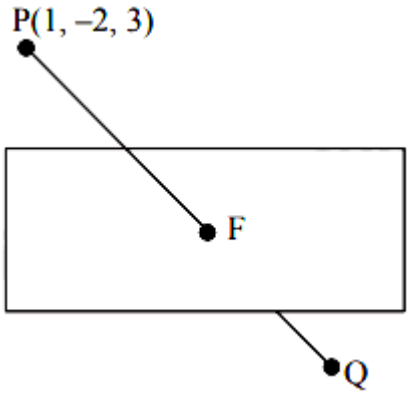
C. $2\sqrt{42}$

D. $\sqrt{42}$

Answer: C

Solution:

Solution:
Equation of line PQ is
$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5}$
Let F be $(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$



Since F lies on the plane
$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$
$2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$
$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$
$\therefore$F is (2,2,8)
$PQ = 2PF = 2\sqrt{1^2 + 4^2 + 5^2} = 2\sqrt{42}$

Question268

The distance of the point (1,3,-7) from the plane passing through the point (1, −1, −1), having normal perpendicular to both the lines

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ **and** $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$**, is :**
[2017]

Options:

A. $\frac{10}{\sqrt{74}}$

B. $\frac{20}{\sqrt{74}}$

C. $\frac{10}{\sqrt{83}}$

D. $\frac{5}{\sqrt{83}}$

Answer: C

Solution:

Solution:
Let the plane be
$a(x-1)+b(y+1)+c(z+1)=0$
Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$
$\Rightarrow 5x+7y+3z+5=0$
Distance of point (1,3,-7) from the plane is
$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$

Question269

If x = a, y = b, z = c is a solution of the system of linear equations x + 8y + 7z = 0 9x + 2y + 3z = 0 x + y + z = 0 such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals:
[Online April 9, 2017]

Options:

A. -1

B. 0

C. 1

D. 2

Answer: C

Solution:

Solution:
$x + 8y + 7z = 0$
$9x + 2y + 3z = 0$
$x + y + z = 0$
$x = \lambda \quad y = 6\lambda \quad z = -7\lambda$
$x = \lambda \quad y = 6\lambda \quad z = -7\lambda$
Now, $\lambda + 12\lambda - 7\lambda = 6$
$6\lambda = 6$
$\lambda = 1$
$\therefore 2\lambda + 6\lambda - 7\lambda$
$= \lambda$
$= 1$

Question270

If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is :
[Online April 9, 2017]

Options:

A. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$

B. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 3$

C. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$

D. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

Answer: A

Solution:

Solution:
Suppose centroid be (h, k, l)
$\therefore x-\text{int}\, p=3h,\ y-\text{int}\, p=3k,\ z-\text{int}\, p=3l$
Equation $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3l}=1$
$\therefore$ Distance from (0,0,0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}\right|=3$$

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$$

Question271

If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x-4y+3z=2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is : [Online April 9, 2017]

Options:

A. 2

B. 1

C. 0

D. 3

Answer: C

Solution:

Solution:
Point $(3, -2, -\lambda)$ on p line $2x-4y+3z-2=0\ \ =6+8-3\lambda-2\ \ =0=3\lambda=12$
$\lambda=4$
Now,
$\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1$ .....(i)

$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2$ .....(ii)

Point on equation (i) $P(k_1+3,-k_1-2,-2k_1-4)$

Point on equation (ii) $Q(12k_2+1,9k_2,4k_2)$

$k_1+3=12k_2+1 \mid -k_1-2=9k_2 \mid -2k_1-4=4k_2$

$k_2=0$

$k_1=-2$

$p(1,0,0)$ lie on equation of a line 1

gives shortest distance $=0$

Question272

The coordinates of the foot of the perpendicular from the point (1,-2,1) on the plane containing the lines, $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is :

[Online April 8, 2017]

Options:

A. (2,-4,2)

B. (-1,2,-1)

C. (0,0,0)

D. (1,1,1)

Answer: C

Solution:

Solution:

$$\vec{n}=\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\6&7&8\\3&5&7\end{vmatrix}=(9,-18,9)=(1,-2,1)$$

$\therefore$ Equation of plane is

$1(x+1)-2(y-1)+(z-3)=0$

$\Rightarrow x-2y+z=0$

foot to z

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-1}{1}=-\frac{[1+4+1]}{6}$

$x=0, y=0, z=0$

Question273

The line of intersection of the planes $\vec{r}\,.\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\vec{r}\,.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$, is :
[Online April 8, 2017]

Options:

A. $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$

B. $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

C. $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

D. $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

Answer: C

Solution:

Solution:

$$\vec{n}=\vec{n_1}\times\vec{n_2}\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-1&1\\1&4&-2\end{vmatrix}=\hat{i}(-2)-\hat{j}(-7)+\hat{k}(13)\Rightarrow\vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$$

Now,
$3x-y+z=1$
$x+4y-2z=2$
but $z=0$& solving the given
$x=6/13$ & $y=5/13$
∴ required equation of a line is
$\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$

Question274

ABC is triangle in a plane with vertices A(2, 3, 5), B(−1, 3,2) and C(λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is:
[Online April 10, 2016]

Options:

A. 1130

B. 1348

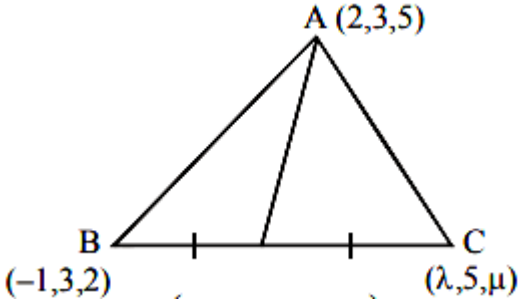
C. 1077

D. 676

Answer: B

Solution:

Solution:
DR's of AD are $\frac{\lambda-1}{2}-2, 4-3, \mu+22-5$
i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$
∵ This median is making equal angles with coordinate axes, therefore,



$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$
$\Rightarrow \lambda = 7 \ \& \ \mu = 10$
$\therefore \lambda^3 + \mu^3 = 5 = 1348$

Question275

The number of distinct real values of lambda for which the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar is :
[Online April 10, 2016]

Options:

A. 2

B. 4

C. 3

D. 1

Answer: C

Solution:

Solution:

Lines are coplanar $\begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$

$\Rightarrow 2(4-\lambda^4)+4(\lambda^2-2)=0$
$\Rightarrow 4-\lambda^4+2\lambda^2-4=0 \Rightarrow \lambda^2(\lambda^2-2)=0$
$\Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

Question276

The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval: [Online April 9, 2016]

Options:

A. (3, 4]

B. (2, 3]

C. [1, 2)

D. [0, 1)

Answer: B

Solution:

Solution:
Shortest distance between two lines
$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by,

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

∴ The shortest distance between given lines are

$$\left|\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -1 & 8 & 4 \end{vmatrix}}{\sqrt{(8-8)^2+(-1-8)^2+(16+2)^2}}\right|$$

$$=\left|\frac{0-36+90}{\sqrt{405}}\right|=\frac{54}{20.1}=2.68$$

Question277

If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $1x+my-z=9$, then l^2+m^2 is equal to:
[2016]

Options:

A. 5

B. 2

C. 26

D. 18

Answer: B

Solution:

Solution:
Line lies in the plane $\Rightarrow(3,-2,-4)$ lie in the plane
$\Rightarrow 3l-2m+4=9$ or $3l-2m=5$ .....(1)
Also, $l, m, -1$ are dr's of line perpendicular to plane and 2 . -1,3 are dr's of line lying in the plane
$\Rightarrow 2l-m-3=0$ or $2l-m=3$ .....(2)

Solving (1) and (2) we get $l = 1$ and $m = -1$
$\Rightarrow l^2 + m^2 = 2$

Question278

The distance of the point (1,-5,9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is:
[2016]

Options:

A. $\frac{10}{\sqrt{3}}$

B. $\frac{20}{3}$

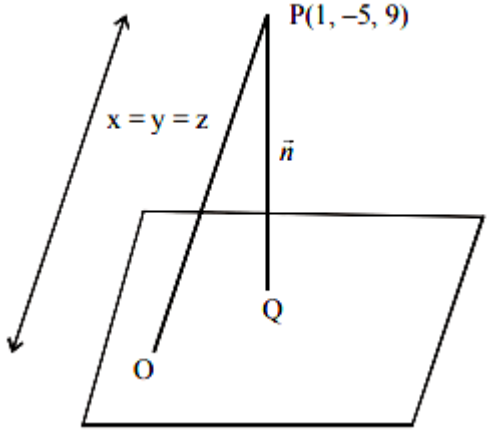
C. $3\sqrt{10}$

D. $10\sqrt{3}$

Answer: D

Solution:

Solution:



Eq^n of PO : $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$
$\Rightarrow x = \lambda + 1; y = \lambda - 5; z = \lambda + 9$
Putting these in eq^n of plane :
$\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$
$\Rightarrow \lambda = -10$
$\Rightarrow$ O is (-9,-15,-1)
$\Rightarrow$ distance $OP = 10\sqrt{3}$

Question279

The distance of the point (1,-2,4) from the plane passing through the point (1,2,2) and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$, is
[Online April 9, 2016]

Options:

A. 2

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Let equation of plane be
$a(x-1)+b(y-2)+c(z-2)=0$ ..... (1)
is perpendicular to given planes then
$a-b+2c=0$
$2a-2b+c=0$
Solving above equation $c=0$ and $a=b$
equation of plane (1) can be
$x+y-3=0$
distance from (1,-2,4) will be
$D=\frac{|1-2-3|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$

Question280

The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is:
[2015]

Options:

A. $x + 3y + 6z = 7$

B. $2x + 6y + 12z = -13$

C. $2x + 6y + 12z = 13$

D. $x + 3y + 6z = -7$

Answer: A

Solution:

Solution:
Equation of the plane containing the lines
$2x - 5y + z = 3$ and $x + y + 4z = 5$ is
$2x - 5y + z - 3 + \lambda\,(x + y + 4z - 5) = 0$
$\Rightarrow (2 + \lambda)x + (-5 + \lambda)y + (1 + 4\lambda)\,z + (-3 - 5\lambda) = 0$
Since the plane (i) parallel to the given plane
$x + 3y + 6z = 1$
$\therefore \frac{2 + \lambda}{1} = \frac{-5 + \lambda}{3} = \frac{1 + 4\lambda}{6}$
$\Rightarrow \lambda = -\frac{11}{2}$
Hence equation of the required plane is
$\left(2 - \frac{11}{2}\right)x + \left(-5 - \frac{11}{2}\right)y + \left(1 - \frac{44}{2}\right)z + \left(-3 + \frac{55}{2}\right) = 0$
$\Rightarrow (4 - 11)x + (-10 - 11)y + (2 - 44)z + (-6 + 55) = 0$
$\Rightarrow -7x - 21y - 42z + 49 = 0$
$\Rightarrow x + 3y + 6z - 7 = 0$
$\Rightarrow x + 3y + 6z = 7$

Question281

The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane$x - y + z = 16$, is

[2015]

Options:

A. $3\sqrt{21}$

B. 13

C. $2\sqrt{14}$

D. 8

Answer: B

Solution:

Solution:
General point on given line $\equiv P(3r+2, 4r-1, 12r+2)$
Point P must satisfy equation of plane
$(3r+2)-(4r-1)+(12r+2)=16$
$11r+5=16$
$r=1$
$P(3\times 1+2, 4\times 1-1, 12\times 1+2) = P(5, 3, 14)$
distance between P and (1,0,2)
$D=\sqrt{(5-1)^2+3^2+(14-2)^2} = 13$

Question282

The shortest distance between the z -axis and the line $x+y+2z-3=0=2x+3y+4z-4$, is
[Online April 11, 2015]

Options:

A. 1

B. 2

C. 4

D. 3

Answer: B

Solution:

Solution:
The equation of any plane passing through given line is
$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$
$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0$
If this plane is parallel to z -axis then normal to the plane will be perpendicular to z-axis.
$\therefore(1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$
$\lambda=-\frac{1}{2}$
Thus, Required plane is

$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0$

$\therefore S.D=\frac{2}{\sqrt{(1)^2}}=2$

Question283

A plane containing the point (3,2,0) and the line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ also contains the point:
[Online April 11, 2015]

Options:

A. (0,3,1)

B. (0,7,-10)

C. (0,-3,1)

D. 0,7,10

Answer: C

Solution:

Solution:
Equation of the plane containing the given line
$\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ is
$A(x-1)+B(y-2)+C(z-3)=0$ .....(i)
where $A+5B+4C=0$ .....(ii)
Since the point (3,2,0) contains in the plane (i), therefore $2A+0.B-3C=0$ .....(iii)
From equations (ii) and (iii),
$\frac{A}{-15-0}=\frac{B}{6+3}=\frac{C}{0-10}=k(\text{ let })$
$\Rightarrow A=-15k, B=9k$ and $C=-10k$
Putting the value of A, B and C in equation (i), we get
$-15(x-1)+9(y-2)-10(z-3)=0$ .....(iv)
Now the coordinates of the point (0,-3,1)
satisfy the equation of the plane (iv) as
$-15(0-1)+9(-3-2)-10(1-3)$
$=15-45+20=0$
Hence the point (0, –3, 1) contains in the plane.

Question284

If the points (1, 1, λ) and (-3,0,1) are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation:
[Online April 10, 2015]

Options:

A. $3x^2 + 10x - 13 = 0$

B. $3x^2 - 10x + 21 = 0$

C. $3x^2 - 10x + 7 = 0$

D. $3x^2 + 10x - 7 = 0$

Answer: C

Solution:

Solution:
$|3 + 4 - 12\lambda + 13| = |-9 + 0 - 12 + 13|$
$\Rightarrow |-12\lambda + 20| = |8| \Rightarrow 3\lambda - 5| = 2$
$\Rightarrow 9\lambda^2 + 25 - 30\lambda = 4 \Rightarrow 9\lambda^2 - 30\lambda + 21 = 0$
$\Rightarrow 3\lambda^2 - 10\lambda + 7 = 0$

Question285

If the shortest distance between the lines $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, $(\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value α is:
[Online April 10, 2015]

Options:

A. $-\frac{16}{19}$

B. $-\frac{19}{16}$

C. $\frac{32}{19}$

D. $\frac{19}{32}$

Answer: C

Solution:

Solution:
Plane passing through $x+y+z+1=0$ and $2x-y+z+3=0$ is $x+y+z+1+\lambda(2x-y+z+3)=0$
$\Rightarrow(2\lambda+1)x+(1-\lambda)y+(1+\lambda)z+3\lambda+1=0$
Parallel to the given line if
$\alpha(2\lambda+1)-1(1-\lambda)+1(1+\lambda)=0$
$\Rightarrow\alpha=\frac{-2\lambda}{2\lambda+1}$..... (i)
Also, $\left|\frac{2\lambda+1-(1-\lambda)+0+3\lambda+1}{\sqrt{(2\lambda+1)^2+(1-\lambda)^2+(1+\lambda)^2}}\right|=\frac{1}{\sqrt{3}}$
$\Rightarrow\lambda=0, \frac{-32}{102}$; $\alpha=0$ or $\alpha=\frac{32}{19}$

Question286

The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2$ is
[2014]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:
Given, $l+m+n=0$ and $l^2=m^2+n^2$
Now, $(-m-n)^2=m^2+n^2$
$\Rightarrow mn=0\Rightarrow m=0$ or $n=0$
If $m=0$ then $l=-n$
We know $l^2+m^2+n^2=1\Rightarrow n=\pm\frac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1) = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

If $n = 0$ then $l = -m$

$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$

$\Rightarrow m = \pm\frac{1}{\sqrt{2}}$

Let $m = \frac{1}{\sqrt{2}}$

$\Rightarrow l = -\frac{1}{\sqrt{2}}$ and $n = 0$

$(l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Question287

Let A(2, 3, 5), B(−1, 3, 2) and C(λ, 5, μ) be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then:
[Online April 11, 2014]

Options:

A. $5\lambda - 8\mu = 0$

B. $8\lambda - 5\mu = 0$

C. $10\lambda - 7\mu = 0$

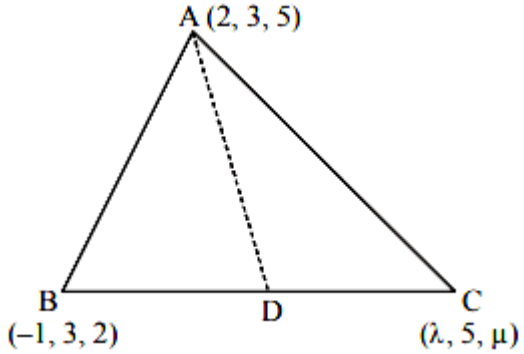
D. $7\lambda - 10\mu = 0$

Answer: C

Solution:

Solution:
If D be the mid-point of BC, then

$D = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$



Direction ratios of AD are $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

Since median AD is equally inclined with coordinate axes, therefore direction ratios of AD will be equal, i.e,

$$\frac{\left(\frac{\lambda-5}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2} = \frac{1}{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}$$

$$= \frac{\left(\frac{\mu-8}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}$$

$$\Rightarrow \left(\frac{\lambda-5}{2}\right)^2 = 1 = \left(\frac{\mu-8}{2}\right)^2$$

$\Rightarrow \lambda = 7, 3$ and $\mu = 10, 6$

If $\lambda = 7$ and $\mu = 10$

Then $\frac{\lambda}{\mu} = \frac{7}{10} \Rightarrow 10\lambda - 7\mu = 0$

Question288

A line in the 3 -dimensional space makes an angle θ $\left(0 < \theta \leq \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of theta is the interval:
[Online April 9, 2014]

Options:

A. $\left(0, \frac{\pi}{4}\right]$

B. $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right]$

Answer: C

Solution:

Solution:
It makes θ with x and y -axes.
$l = \cos\theta, m = \cos\theta, n = \cos(\pi - 2\theta)$
we have $l^2 + m^2 + n^2 = 1$
$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2(\pi - 2\theta) = 1$
$\Rightarrow 2\cos^2\theta + (-\cos 2\theta)^2 = 1$
$\Rightarrow 2\cos^2\theta - 1 + \cos^2 2\theta = 0$
$\Rightarrow \cos 2\theta - [1 + \cos 2\theta] = 0$
$\Rightarrow \cos 2\theta = 0$ or $\cos 2\theta = -1$
$\Rightarrow 2\theta = \pi/2$ or $2\theta = \pi$
$\Rightarrow \theta = \pi/4$ or $\theta = \frac{\pi}{2}$
$\Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Question289

Equation of the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ is:
[Online April 19, 2014]

Options:

A. $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-2}$

B. $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$

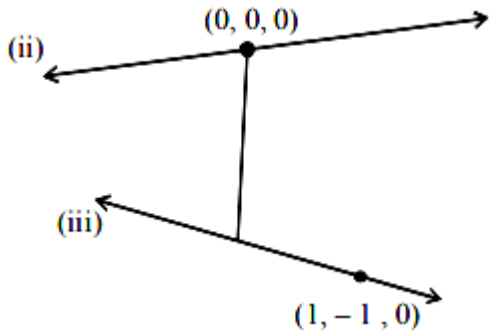
C. $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{1}$

D. $\frac{x}{-2} = \frac{y}{1} = \frac{z}{2}$

Answer: B

Solution:

Solution:
Let equation of the required line be
$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ .....(i)
Given two lines
$\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ .....(ii)
and $\frac{x-1}{0}=\frac{y+1}{0}=\frac{z}{1}$ .....(iii)
Since the line (i) is perpendicular to both the lines (ii) and (iii), therefore
$a-b+c=0$ .....(iv)
$-2b+c=0$ .....(v)
From (iv) and (v) $c=2b$ and $a+b=0$, which are not satisfy by options (c) and (d). Hence options (c) and (d) are rejected.
Thus point (x_1, y_1, z_1) on the required line will be either (0,0,0) or (1,-1,0).
Now foot of the perpendicular from point (0,0,0) to the line(iii) $=(1,-2r-1,r)$



The direction ratios of the line joining the points (0,0,0) and $(1,-2r-1,r)$ are $1,-2r-1,r$
Since sum of the x and y -coordinate of direction ratio of the required line is 0 .
$\therefore 1-2r-1=0, \Rightarrow r=0$
Hence direction ratio are 1,-1,0
But the z-direction ratio of the required line is twice the y -direction ratio of the required line
i.e. $0=2(-1)$, which is not true.
Hence the shortest line does not pass through the point (0,0,0) . Therefore option (a) is also rejected.

Question290

The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane$2x-y+z+3=0$ is the line:
[2014]

Options:

A. $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

B. $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

C. $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

D. $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

Answer: C

Solution:

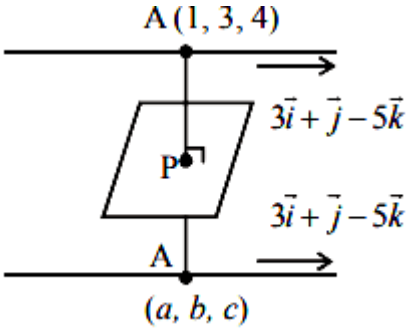
Solution:

$\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$(let)

$a=2\lambda+1$

$b=3-\lambda$

$c=4+\lambda$



$P=\left(\frac{a+1}{2},\frac{b+3}{2},\frac{c+4}{2}\right)=\left(\lambda+1,\frac{6-\lambda}{2},\frac{\lambda+8}{2}\right)$

$\therefore 2(\lambda+1)-\frac{6-\lambda}{2}+\frac{\lambda+8}{2}+3=0$

$3\lambda+6=0\Rightarrow\lambda=-2$

$a=-3, b=5, c=2$

Required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

Question291

If the angle between the line $2(x+1)=y=z+4$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is $\frac{\pi}{6}$, then the value of λ is: [Online April 19, 2014]

Options:

A. $\frac{135}{7}$

B. $\frac{45}{11}$

C. $\frac{45}{7}$

D. $\frac{135}{11}$

Answer: C

Solution:

Solution:
Given equation of line can be written as
$\frac{x+1}{1}=\frac{y}{2}=\frac{z+4}{2}$
Eqn of plane is $2x-y+\sqrt{\lambda}z+4=0$
Since, angle between the line and the plane is $\frac{\pi}{6}$
therefore
$\sin\frac{\pi}{6}=\frac{2(1)+2(-1)+2(\sqrt{\lambda})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$
$\frac{1}{2}=\frac{2-2+2\sqrt{\lambda}}{\sqrt{9}\sqrt{5+\lambda}}$
$\Rightarrow\frac{\sqrt{\lambda}}{\sqrt{5+\lambda}}=\frac{3}{4}\Rightarrow\frac{\lambda}{5+\lambda}=\frac{9}{16}$
$\Rightarrow 7\lambda=45\Rightarrow\lambda=\frac{45}{7}$

Question292

If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is: [Online April 12, 2014]

Options:

A. 41 or -42

B. 42 or -43

C. -41 or 43

D. -42 or 44

Answer: C

Solution:

Solution:
Given planes are
$4x-2y-4z+1=0$
and $4x-2y-4z+d=0$
They are parallel.
Distance between them is $\pm 7=\frac{d-1}{\sqrt{16+4+16}}$
$\Rightarrow \frac{d-1}{6}=\pm 7 \Rightarrow d=42+1$
or $-42+1$ i.e. $d=-41$ or 43 .

Question293

A symmetrical form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is [Online April 12, 2014]

Options:

A. $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$

B. $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$

C. $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$

D. $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

Answer: B

Solution:

Solution:
Given two planes:
$x-ay-b=0$ and $cy-z+d=0$
Let, l, m, n be the direction ratio of the required line. Since the required line is perpendicular to normal of both the plane, therefore $l-am=0$ and $cm-n=0$
$\Rightarrow l-am+0 . n=0$ and $0 . l+cm-n=0$
$\therefore \frac{l}{a-0}=\frac{m}{0+1}=\frac{n}{c-0}$
Hence, $d . R$ of the required line are $a, 1, c$.
Hence, options (c) and (d) are rejected.
Now, the point $(a+b, 1, c+d)$ satisfy the equation of the two given planes.
∴ Option(b) is correct.

Question294

The plane containing the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point:
[Online April 11, 2014]

Options:

A. (1,-2,5)

B. (1,0,5)

C. (0,3,-5)

D. (-1,-3,0)

Answer: B

Solution:

Solution:
Equation of the plane containing the line
$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ is
$a(x-1)+b(y-2)+c(z-3)=0$ .....(i)
where $a.1+b.2+c.3=0$
i.e., $a+2b+3c=0$ .....(ii)
Since the plane (i) parallel to the line
$\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$
$\therefore a.1+b.1+c.4=0$
i.e., $a+b+4c=0$ ..... (iii)
From (ii) and (iii) ,
$\frac{a}{8-3}=\frac{b}{3-4}=\frac{c}{1-2}=k$ (let)
$\therefore a=5k, b=-k, c=-k$
On putting the value of a, b and c in equation (i),
$5(x-1)-(y-2)-(z-3)=0$
$\Rightarrow 5x-y-z=0$ .....(iv)
when $x=1, y=0$ and $z=5$; then
L.H.S. of equation (iv) $=5x-y-2$
$=5\times 1-0-5=0$
$=R.H.S.$ of equation
Hence coordinates of the point (1, 0, 5) satisfy the equation plane represented by equations (iv),
Therefore the plane passes through the point (1,0,5)

Question295

Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is:
[Online April 9, 2014]

Options:

A. $7x+2y+4z=54$

B. $3x+4y+5z=49$

C. $4x+3y+5z=50$

D. $5x+4y+3z=57$

Answer: C

Solution:

Solution:
Given equation of lines are
$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ .....(1)
and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ .....(2)
Any point on line (1) is $P(3\lambda+1, \lambda+2, 2\lambda+3)$ and on line (2) is $Q(\mu+3, 2\mu+1, 3\mu+2)$
On solving $3\lambda+1=\mu+3$ and $\lambda+2=2\mu+1$
we get $\lambda=1, \mu=1$
$\therefore$ Point of intersection of two lines is $R(4, 3, 5)$
So, equation of plane $\perp$ to OR where O is (0,0,0) and passing through R is
$4x+3y+5z=50$

Question296

Let ABC be a triangle with vertices at points A(2, 3, 5), B(−1, 3, 2) and C(λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to :
[Online April 25, 2013]

Options:

A. (10,7)

B. (7,5)

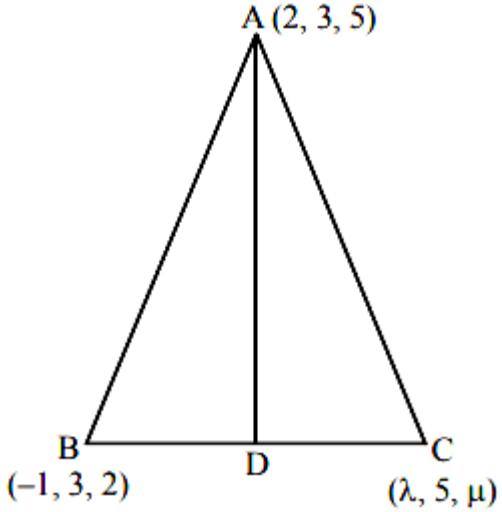
C. (7,10)

D. (5,7)

Answer: C

Solution:

Solution:
Since AD is the median



$\therefore D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

Now, d R 's of AD is

$a = \left(\frac{\lambda-1}{2} - 2\right) = \frac{\lambda-5}{2}$

$b = 4-3 = 1,\ c = \frac{\mu+2}{2} - 5 = \frac{\mu-8}{2}$

Also, a, b, c are dR's

$\therefore a = kl,\ b = km,\ c = kn$ where $l = m = n$

and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

Now, $a = 1,\ b = 1$ and $c = 1$

$\Rightarrow \lambda = 7$ and $\mu = 10$

Question297

If the protections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the

line segment is :
[Online April 23, 2013]

Options:

A. 12

B. 7

C. 9

D. 6

Answer: B

Solution:

Solution:
Length of the line segment $= \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

Question298

The acute angle between two lines such that the direction cosines l , m, n, of each of them satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is
[Online April 22, 2013]

Options:

A. 15°

B. 30°

C. 60°

D. 45°

Answer: C

Solution:

Solution:

Let l_1, m_1, n_1 and l_2, m_2, n_2 be the d.c of line 1 and 2 respectively, then as given

$l_1 + m_1 + n_1 = 0$

and $l_2 + m_2 + n_2 = 0$

and $l_1^2 + m_1^2 - n_1^2 = 0$ and

$l_2^2 + m_2^2 - n_2^2 = 0$

$(\because l + m + n = 0$ and $l^2 + m^2 - n^2 = 0)$

Angle between lines, θ is

$\cos theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ .....(1)

As given $l^2 + m^2 = n^2$ and $l + m = -n$

$\Rightarrow (-n)^2 - 2l m = n^2 \Rightarrow 2l m = 0$ or $l m = 0$

So $l_1 m_1 = 0, l_2 m_2 = 0$

If $l_1 = 0, m_1 \neq 0$ then $l_1 m_2 = 0$

If $m_1 = 0, l_1 \neq 0$ then $l_2 m_1 = 0$

If $l_2 = 0, m_2 \neq 0$ then $l_2 m_1 = 0$

If $m_2 = 0, l_2 \neq 0$ then $l_1 m_2 = 0$

Also $l_1 l_2 = 0$ and $m_1 m_2 = 0$

$l^2 + m^2 - n^2 = l^2 + m^2 + n^2 - 2n^2 = 0$

$\Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore n_1 = \pm\frac{1}{\sqrt{2}}, n_2 = \pm\frac{1}{\sqrt{2}}$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$(acute angle)

Question299

If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have
[2013]

Options:

A. any value

B. exactly one value

C. exactly two values

D. exactly three values

Answer: C

Solution:

Solution:
Given lines will be coplanar

If $\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow -1(1+2k) - (1+k^2) + 1(2-k) = 0$
$\Rightarrow k = 0, -3$

Question300

If two lines L_1 and L_2 in space, are defined by$L_1 = \{x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1), z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\}$ and $L_2 = \{x = \sqrt{\mu}y + (1 - \sqrt{\mu}), z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$ then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that : [Online April 23, 2013]

Options:

A. $\sqrt{\lambda} + \sqrt{\mu} = 1$

B. $\lambda \neq \mu$

C. $\lambda + \mu = 0$

D. $\lambda = \mu$

Answer: D

Solution:

Solution:
For L_1,

$x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1) \Rightarrow y = \frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}}$ .....(i)

$z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda} \Rightarrow y = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ .....(ii)

From (i) and (ii)

$\frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = \frac{y - 0}{1} = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ .....(A)

The equation (A) is the equation of line L_1.

Similarly equation of line L_2 is

$$\frac{x-(1-\sqrt{\mu})}{\sqrt{\mu}}=\frac{y-0}{1}=\frac{z-\sqrt{\mu}}{1-\sqrt{\mu}} \text{ .....(B)}$$

Since $L_1 \perp L_2$, therefore

$$\sqrt{\lambda}\sqrt{\mu}+1\times 1+(\sqrt{\lambda}-1)(1-\sqrt{\mu})=0$$

$$\Rightarrow \sqrt{\lambda}+\sqrt{\mu}=0 \Rightarrow \sqrt{\lambda}=-\sqrt{\mu}$$

$$\Rightarrow \lambda=\mu$$

Question301

If the lines $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z+1}{3}$ and $\frac{x+2}{2}=\frac{y-k}{3}=\frac{z}{4}$ are coplanar, then the value of k is : [Online April 9, 2013]

Options:

A. $\frac{11}{2}$

B. $-\frac{11}{2}$

C. $\frac{9}{2}$

D. $-\frac{9}{2}$

Answer: A

Solution:

Solution:
Two given planes are coplanar, if

$$\begin{vmatrix} -2-(-1) & k-1 & 0-(-1) \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} -1 & k-1 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0$$

$$\Rightarrow (-1)(4-9)-(k-1)(8-6)+6-2=0$$

$$\Rightarrow k=\frac{11}{2}$$

Question302

Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
[2013]

Options:

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

Answer: C

Solution:

Solution:
$2x + y + 2z - 8 = 0$ ....(Plane 1)
$2x + y + 2z + \frac{5}{2} = 0$ ....(Plane 2)Distance between Plane 1 and 2

$$= \left| \frac{-8 - \frac{5}{2}}{\sqrt{2^2 + 1^2 + 2^2}} \right| = \left| \frac{-21}{6} \right| = \frac{7}{2}$$

Question303

The equation of a plane through the line of intersection of the planes $x + 2y = 3$, $y - 2z + 1 = 0$, and perpendicular to the first plane is:
[Online April 25, 2013]

Options:

A. $2x - y - 10z = 9$

B. $2x - y + 7z = 11$

C. $2x - y + 10z = 11$

D. $2x - y - 9z = 10$

Answer: C

Solution:

Solution:
Equation of a plane through the line of intersection of the planes
$x + 2y = 3, y - 2z + 1 = 0$ is
$(x + 2y - 3) + \lambda(y - 2z + 1) = 0$
$\Rightarrow x + (2 + \lambda)y - 2\lambda(z) - 3 + \lambda = 0$ .....(i)
Now, plane (i) is $\perp$ to $x + 2y = 3$
$\therefore$ Their dot product is zero
i.e. $1 + 2(2 + \lambda) = 0 \Rightarrow \lambda = -\frac{5}{2}$
Thus, required plane is
$x + \left(2 - \frac{5}{2}\right)y - 2 \times \frac{-5}{2}(z) - 3 - \frac{5}{2} = 0$
$\Rightarrow x - \frac{y}{2} + 5z - \frac{11}{2} = 0$
$\Rightarrow 2x - y + 10z - 11 = 0$

Question304

Let Q be the foot of perpendicular from the origin to the plane $4x - 3y + z + 13 = 0$ and R be a point (-1,-6) on the plane. Then length QR is:
[Online April 22, 2013]

Options:

A. $\sqrt{14}$

B. $\sqrt{\frac{19}{2}}$

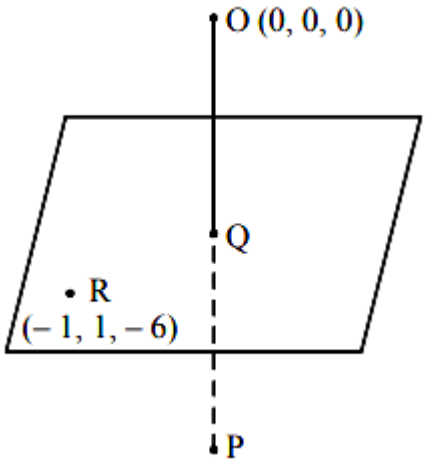
C. $3\sqrt{\frac{7}{2}}$

D. $\frac{3}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Let P be the image of O in the given plane.



Equation of the plane, $4x-3y+z+13=0$
OP is normal to the plane, therefore direction ratio of OP are proportional to 4,-3,1
Since OP passes through (0,0,0) and has direction ratio proportional to $4,-3,1$. Therefore equation of OP is

$$\frac{x-0}{4}=\frac{y-0}{-3}=\frac{z-0}{1}=r \text{ (let)}$$

$$\therefore x=4r,\ y=-3r,\ z=r$$

Let the coordinate of P be $(4r,-3r,r)$
Since Q be the mid point of OP

$$\therefore Q=\left(2r,-\frac{3}{2}r,\frac{r}{2}\right)$$

Since Q lies in the given plane

$$4x-3y+z+13=0$$

$$\therefore 8r+\frac{9}{2}r+\frac{r}{2}+13=0$$

$$\Rightarrow r=\frac{-13}{8+\frac{9}{2}+\frac{1}{2}}=\frac{-26}{26}=-1$$

$$\therefore Q=\left(-2,\frac{3}{2},-\frac{1}{2}\right)$$

$$QR=\sqrt{(-1+2)^2+\left(1-\frac{3}{2}\right)^2+\left(-6+\frac{1}{2}\right)^2}$$

$$=\sqrt{1+\frac{1}{4}+\frac{121}{4}}=3\sqrt{\frac{7}{2}}$$

Question305

A vector $\vec{n}$ is inclined to x -axis at 45°, to y -axis at 60° and at an acute angle to z -axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2},-1,1)$ then the equation of the plane is:
[Online April 9, 2013]

Options:

A. $4\sqrt{2}x + 7y + z - 2$

B. $2x + y + 2z = 2\sqrt{2} + 1$

C. $3\sqrt{2}x - 4y - 3z = 7$

D. $\sqrt{2}x - y - z = 2$

Answer: B

Solution:

Solution:
Direction cosines of $\vec{n}$ are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.
Equation of the plane,
$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{4}(y + 1) + \frac{1}{2}(z - 1) = 0$
$\Rightarrow 2(x - \sqrt{2}) + (y + 1) + 2(z - 1) = 0$
$\Rightarrow 2x + y + 2z = 2\sqrt{2} - 1 + 2$
$\Rightarrow 2x + y + 2z = 2\sqrt{2} + 1$

Question306

If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to:
[2012]

Options:

A. -1

B. $\frac{2}{9}$

C. $\frac{9}{2}$

D. 0

Answer: C

Solution:

Solution:

Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$

and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$

$\therefore \vec{a}_1=\hat{i}-\hat{j}+\hat{k},\ \vec{b}_1=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{a}_2=3\hat{i}+k\hat{j},\ \vec{b}_2=\hat{i}+2\hat{j}+\hat{k}$

Given lies are intersect if

$$\frac{\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}=0$$

$$\Rightarrow\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)=0$$

$$\Rightarrow\begin{vmatrix}2 & k+1 & -1\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$$

$\Rightarrow 2(3-8)-(k+1)(2-4)-1(4-3)=0$

$\Rightarrow 2(-5)-(k+1)(-2)-1(1)=0$

$\Rightarrow -10+2k+2-1=0\Rightarrow k=\frac{9}{2}$

Question307

The distance of the point $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line that passes through the point $2\hat{i}+3\hat{j}-4\hat{k}$ and is parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$ is
[Online May 26, 2012]

Options:

A. 9

B. 8

C. 7

D. 10

Answer: C

Solution:

Solution:
Point is (–1, 2, 6)
Line passes through the point (2, 3, –4) parallel to vector whose direction ratios is 6, 3, – 4.

Equation is $\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}=\lambda$
Any point on this line is given by $x=6\lambda+2,\ y=3\lambda+3,\ z=-4\lambda-4$
Now, d .Rs of line passing through (-1,2,6) and $\perp$ to this line is $\{(x+1),(y-2),(z-6)\}$
So, $6(x+1)+3(y-2)-4(z-6)=0$
$\Rightarrow 6x+3y-4z+24=0$
Now, $6(6\lambda+2)+3(3\lambda+3)+4(4\lambda+4)+24=0$
$\Rightarrow 61\lambda+61=0\Rightarrow\lambda=-1$
So, $x=-4,\ y=0,\ z=0$
Now, distance between (-1,2,6) and (-4,0,0) is $\sqrt{9+4+36}=\sqrt{49}=7$

Question308

Statement 1: The shortest distance between the lines $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and $\frac{x-1}{4}=\frac{y-1}{-2}=\frac{z-1}{4}$ is $\sqrt{2}$.

Statement 2 : The shortest distance between two parallel lines is the perpendicular distance from any point on one of the lines to the other line.

[Online May 19, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

C. Statement 1 is false, Statement 2 is true.

D. Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1

Answer: C

Solution:

Solution:
On solving we will get shortest distance $\neq\sqrt{2}$

Question309

The coordinates of the foot of perpendicular from the point (1, 0, 0) to the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are
[Online May 12, 2012]

Options:

A. $(2, -3, 8)$

B. $(1, -1, -10)$

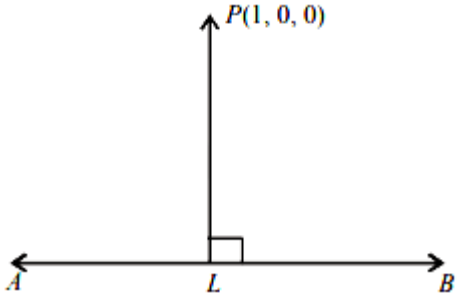
C. $(5, -8, -4)$

D. $(3, -4, -2)$

Answer: D

Solution:

Solution:
Let the equation of AB is
$\frac{x-1}{2}=\frac{y-(-1)}{-3}=\frac{z-(-10)}{8}=k$
Let L be the foot of the perpendicular drawn from $P(1, 0, 0)$



Now, direction ratio of $PL = (2k, -3k-1, 8k-10)$ and direction ratio of $AB = (2, -3, 8)$
Since, PL is perpendicular to AB
$\therefore 2(2k)-3(-3k-1)+8(8k-10)=0$
Now, $k=\frac{2(1-1)+(-3)(0+1)+8(0+10)}{(2)^2+(-3)^2+(8)^2}$
$=\frac{0-3+80}{4+9+64}=\frac{77}{77}=1$
$\therefore$ Required co-ordinate $= L = (2+1, -3-1, 8-10)$
$= (3, -4, -2)$

Question310

A equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is :

[2012]

Options:

A. $x - 2y + 2z - 3 = 0$

B. $x - 2y + 2z + 1 = 0$

C. $x - 2y + 2z - 1 = 0$

D. $x - 2y + 2z + 5 = 0$

Answer: A

Solution:

Solution:
Given that, equation of a plane is
$x - 2y + 2z - 5 = 0$
So, Equation of parallel plane is
$x - 2y + 2z + d = 0$
Now, it is given that distance from origin to the parallel plane is 1 .
$\therefore \left| \frac{d}{\sqrt{1^2 + 2^2 + 2^2}} \right| = 1 \Rightarrow d = \pm 3$
So equation of required plane $x - 2y + 2z \pm 3 = 0$

Question311

The equation of a plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0,7,-7) is
[Online May 26, 2012]

Options:

A. $x + y + z = 0$

B. $x + 2y + z = 21$

C. $3x - 2y + 5z + 35 = 0$

D. $3x + 2y + 5z + 21 = 0$

Answer: A

Solution:

Solution:
The equation of the plane containing the line
$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is a $(x+1)+b(y-3)+c\,(z+2)=0$
where $-3a+2b+c=0$ .....(A)
This passes through (0,7,-7)
$\therefore a(0+1)+b(7-3)+c(-7+2)=0$
$\Rightarrow a+4b-5c=0$.....(B)
On solving equation (A) and (B) we get
$a=1, b=1, c=1$
$\therefore$ Required plane is
$x+1+y-3+z+2=0$
$\Rightarrow x+y+z=0$

Question312

Consider the following planes
P : x + y − 2z + 7 = 0
Q : x + y + 2z + 2 = 0
R : 3x + 3y − 6z − 11 = 0
[Online May 26, 2012]

Options:

A. P and R are perpendicular

B. Q and R are perpendicular

C. P and Q are parallel

D. P and R are parallel

Answer: D

Solution:

Solution:
Given planes are
$P: x+y-2z+7=0$
$Q: x+y+2z+2=0$
and $R: 3x+3y-6z-11=0$
Consider Plane P and R.
Here $a_1=1, b_1=1, c_1=-2$
and $a_2=3, b_2=3, c_2=-6$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$
therefore P and R are parallel.

Question313

If the three planes $x = 5$, $2x - 5ay + 3z - 2 = 0$ and $3bx + y - 3z = 0$ contain a common line, then (a, b) is equal to [Online May 19, 2012]

Options:

A. $\left(\frac{8}{15}, -\frac{1}{5}\right)$

B. $\left(\frac{1}{5}, -\frac{8}{15}\right)$

C. $\left(-\frac{8}{15}, \frac{1}{5}\right)$

D. $\left(-\frac{1}{5}, \frac{8}{15}\right)$

Answer: B

Solution:

Solution:
Let the direction ratios of the common line be l, m and n.
$\therefore l \times 1 + m \times 0 + n \times 0 = 0 \Rightarrow l = 0$
$2l - 5ma + 3n = 0 \Rightarrow 5ma - 3n = 0$
$3l\,b + m - 3n = 0 \Rightarrow m - 3n = 0$ .....(3)
Subtracting (3) from (1), we get $m(5a - 1) = 0$
Now, value of m can not be zero because if $m = 0$ then $n = 0$
$\Rightarrow l = m = n = 0$ which is not possible.
Hence, $5a - 1 = 0 \Rightarrow a = \frac{1}{5}$
Thus, option (b) is correct.

Question314

A line with positive direction cosines passes through the point P(2, −1, 2) and makes equal angles with the coordinate axes. If the line meets the plane 2x + y + z = 9 at point Q, then the length PQ equals
[Online May 7, 2012]

Options:

A. $\sqrt{2}$

B. 2

C. $\sqrt{3}$

D. 1

Answer: C

Solution:

Solution:
Point P is (2,-1,2)
Let this line meet at $Q(h, k, w)$
Direction ratio of this line is
$(h-2, k+1, w-2)$
Since, $d c_s$ are equal $\& d r_s$ are also equal,
So, $h-2=k+1+w-2$
$\Rightarrow k=h-3$ and $w=h$
This line meets the plane
$2x+y+z=9$ at Q, so,
$2h+k+w=9$ or $2h+h-3+h=9$
$\Rightarrow 4h-3=9 \Rightarrow h=3$
and $k=0$ and $w=3$
$\text{DistancePQ} = \sqrt{(3-2)^2+(0-(-1))^2+(3-2)^2}$
$= \sqrt{1^2+1^2+1^2} = \sqrt{3}$

Question315

The values of a for which the two points (1, a, 1) and (−3, 0, a) lie on the opposite sides of the plane 3x + 4y− 12z + 13 = 0, satisfy
[Online May 7, 2012]

Options:

A. $0 < a < \frac{1}{3}$

B. $-1 < a < 0$

C. $a < -1$ or $a < \frac{1}{3}$

D. $a = 0$

Answer: D

Solution:

Solution:
Given equation of plane is
$3x + 4y - 12z + 13 = 0$
$(1, a, 1)$ and $(-3, 0, a)$ satisfy the equation of plane.
$\therefore$ We have
$3 + 4(a) - 12 + 13 = 0$ and $3(-3) - 12(a) + 13 = 0$
$\Rightarrow 4 + 4a = 0$ and $4 - 12a = 0$
$\Rightarrow a = -1$ and $a = \frac{1}{3}$
Since, $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $\therefore a = 0$

Question316

The length of the perpendicular drawn from the point(3,-1,11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:
[2011RS]

Options:

A. $\sqrt{29}$

B. $\sqrt{33}$

C. $\sqrt{53}$

D. $\sqrt{66}$

Answer: C

Solution:

Solution:
Any point on line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\alpha$ is $(2\alpha, 3\alpha+2, 4\alpha+3)$
$\Rightarrow$ Direction ratio of the $\perp$ line is
$2\alpha-3, 3\alpha+3, 4\alpha-8.$
and Direction ratio of the given line are 2,3,4
$\Rightarrow 2(2\alpha-3)+3(3\alpha+3)+4(4\alpha-8)=0$
$\Rightarrow 29\alpha-29=0$
$\Rightarrow \alpha=1$
$\Rightarrow$ Foot of $\perp$ is (2,5,7)
$\Rightarrow$ Length $\perp$ is $\sqrt{1^2+6^2+4^2}=\sqrt{53}$

Question317

Statement-1: The point A(1, 0, 7)) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

[2011]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:
The direction ratio of the line segment AB is 0,6,-4 and the direction ratio of the given line is 1,2,3 .
Clearly $1 \text{ times } 0+2\times 6+3\times(-4)=0$
So, the given line is perpendicular to line AB.
Also, the mid point of A and B is (1,3,5) which satisfy the given line.
So, the image of B in the given line is A statement- 1 and 2 both true but 2 is not correct explanation. of 1 .

Question318

The distance of the point (1,-5,9) from the plane x − y+ z = 5 measured along a straight x = y = z is
[2011RS]

Options:

A. $10\sqrt{3}$

B. $5\sqrt{3}$

C. $3\sqrt{10}$

D. $3\sqrt{5}$

Answer: A

Solution:

Solution:
Equation of line through $P(1, -5, 9)$ and parallel to the line $x = y = z$ is
$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda(\text{say})$
$Q = (x = 1+\lambda, y = -5+\lambda, z = 9+\lambda)$
Since Q lies on plane $x - y + z = 5$
$\therefore 1+\lambda+5-\lambda+9+\lambda = 5$
$\Rightarrow \lambda = -10$
$\therefore Q = (-9, -15, -1)$
$\therefore PQ = \sqrt{(1+9)^2+(15-5)^2+(9+1)^2}$
$= \sqrt{300} = 10\sqrt{3}$

Question319

If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals
[2011]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{5}$

C. $\frac{5}{3}$

D. $\frac{2}{3}$

Answer: D

Solution:

Solution:
Let θ be the angle between the given line and plane, then

$$\sin\theta = \frac{1\times 1+2\times 2+\lambda\times 3}{\sqrt{1^2+2^2+\lambda^2}\,.\,\sqrt{1^2+2^2+3^2}} = \frac{5+3\lambda}{\sqrt{14}\,.\,\sqrt{5+\lambda^2}}$$

$$\Rightarrow\cos\theta = \sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\Rightarrow \sqrt{\frac{5}{14}} = \sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

Squaring both sides, we get

$$\frac{5}{14} = \frac{5\lambda^2-30\lambda+45}{14(5+\lambda^2)}$$

$$\Rightarrow\lambda = \frac{2}{3}$$

Question320

A line AB in three-dimensional space makes angles 45° and 120° with the positive x -axis and the positive y -axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals [2010]

Options:

A. 45°

B. 60°

C. 75°

D. 30°

Answer: B

Solution:

Solution:

As per question, direction cosines of the line : $l = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$, $n = \cos\theta$

where theta is the angle, which line makes with positive z -axis.

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{2}$ (θ being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

Question321

The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13,32) . The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is
[2010]

Options:

A. $\sqrt{17}$

B. $\frac{17}{\sqrt{15}}$

C. $\frac{23}{\sqrt{17}}$

D. $\frac{23}{\sqrt{15}}$

Answer: C

Solution:

Solution:

Slope of line $L = -\frac{b}{5}$

Slope of line $K = -\frac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

(13,32) is a point on L.

$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$

Equation of K :

$y - 4x = 3 \Rightarrow 4x - y + 3 = 0$

Distance between L and $K = \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

Question322

Statement -1 : The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5.
Statement -2: The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).
[2010]

Options:

A. Statement -1 is true, Statement -2 is true ; Statement - 2 is not a correct explanation for Statement -1.

B. Statement -1 is true, Statement -2 is false.

C. Statement -1 is false, Statement -2 is true .

D. Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

Answer: A

Solution:

Solution:

$A(3, 1, 6)$; $B = (1, 3, 4)$

Putting coordinate of mid-point of $AB = (2, 2, 5)$ in plane $x - y + z = 5$ then $2 - 2 + 5 = 5$, satisfy

So, mid-point of $AB = (2, 2, 5)$ lies on the plane.

d.r's of $AB = (2, -2, 2)$

d.r's of normal to plane $= (1, -1, 1)$

Direction ratio of AB and normal to the plane are proportional therefore,
AB is perpendicular to the normal of plane
$\therefore$ A is image of B
Statement-1 is correct.
Statement-2 is also correct but it is not correct explanation.

Question323

The projections of a vector on the three coordinate axis are 6,-3,2 respectively. The direction cosines of the vector are
[2009]

Options:

A. $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

B. $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

C. $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$

D. $6, -3, 2$

Answer: B

Solution:

Solution:
Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6,-3,2 then
$x_2 - x_1, = 6; y_2 - y_1 = -3; z_2 - z_1 = 2$
So that direction ratios of $\overrightarrow{PQ}$ are 6,-3,2
$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are $\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}} \frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

Question324

Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane$x + 3y - \alpha z + \beta = 0$. Then (α, β) equals
[2009]

Options:

A. (-6,7)

B. (5,-15)

C. (-5,5)

D. (6,-17)

Answer: A

Solution:

Solution:
Given that, the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$
$\therefore Pt(2,1,-2)$ lies on the plane
i.e. $2+3+2\alpha+\beta=0$
or $2\alpha+\beta+5=0$ .....(i)
Also normal to plane will be perpendicular to line,
$\therefore 3\times 1-5\times 3+2\times(-\alpha)=0$
$\Rightarrow \alpha=-6$
From equation (i) then, $\beta=7$
$\therefore(\alpha,\beta)=(-6,7)$

Question325

If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to
[2008]

Options:

A. -5

B. 5

C. 2

D. -2

Answer: A

Solution:

Solution:
hen the two lines intersect then shortest distance between them is zero i.e.

$$\frac{\left(\vec{a}_2-\vec{a}_1\right).\vec{b}_1\times\vec{b}_2}{\left|\vec{b}_1\times\vec{b}_2\right|}=0$$

$$\Rightarrow\left(\vec{a}_2-\vec{a}_1\right).\vec{b}_1\times\vec{b}_2=0$$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k},\ \vec{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$$\Rightarrow\begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0$$

$\Rightarrow 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow -2k^2-5k+25=0\Rightarrow k=-5$ or $\frac{5}{2}$

$\because k$ is an integer, therefore $k=-5$

Question326

The line passing through the points (5, 1, a) and (3, b, 1)crosses the yz-plane at the point $\left(0,\frac{17}{2},\frac{-13}{2}\right)$. Then

[2008]

Options:

A. $a=2, b=8$

B. $a=4, b=6$

C. $a=6, b=4$

D. $a=8, b=2$

Answer: C

Solution:

Solution:

Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is $\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$

$x=-2\lambda+5$

$y=(b-1)\lambda+1$

$z=(1-a)\lambda+a$

$\therefore$ Any point on this line is a
$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$
Given that it crosses yz plane $\therefore -2\lambda+5=0$
$\lambda=\frac{5}{2}$
$\therefore \left(0, (b-1)\frac{5}{2}+1, (1-a)\frac{5}{2}+a\right) = \left(0, \frac{17}{2}, \frac{-13}{2}\right)$
$\Rightarrow (b-1)\frac{5}{2}+1=\frac{17}{2}$
and $(1-a)\frac{5}{2}+a=-\frac{13}{2}$
$\Rightarrow b=4$ and $a=6$

Question327

If a line makes an angle of $\pi/4$ with the positive directions of each of x - axis and y -axis, then the angle that the line makes with the positive direction of the z -axis is
[2007]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

Solution:
Let the line makes an angle theta with the positive direction of z -axis. Given that lines makes angle $\frac{\pi}{4}$ with x axis and y -axis.
$\therefore l=\cos\frac{\pi}{4}, m=\cos\frac{\pi}{4}, n=\cos\theta$
We know that ,$l^2+m^2+n^2=1$
$\therefore \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1$

$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Hence, angle with positive direction of the z -axis is $\frac{\pi}{2}$.

Question328

Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x -axis, then $\cos\alpha$ equals
[2007]

Options:

A. 1

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2}$.

Answer: C

Solution:

Solution:
Let the direction cosines of line L be l, m, n. Since line L lies on both planes.
$\therefore 2l + 3m + n = 0$ .....(i)
and $l + 3m + 2n = 0$ .....(ii)
on solving equation (i) and (ii), we get
$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \Rightarrow \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$
Now $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$
$\because l^2 + m^2 + n^2 = 1$
$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$
$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$

Line L, makes an angle α with + ve x -axis

$\therefore 1 = \cos\alpha \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Question329

TOPIC 4 - Sphere and Miscellaneous Problems on Sphere

If (2,3,5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are

[2007]

Options:

A. (4,3,5)

B. (4,3,-3)

C. (4,9,-3)

D. (4,-3,3)

Answer: C

Solution:

Solution:
We know that centre of sphere
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$
is $(-u, -v, -w)$
Given that, $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$
$\therefore$ Centre $\equiv (3, 6, 1)$
Coordinates of one end of diameter of the sphere are (2,3,5)
Let the coordinates of the other end of diameter are(α, β, γ)
$\therefore \frac{\alpha + 2}{2} = 3, \frac{\beta + 3}{2} = 6, \frac{\gamma + 5}{2} = 1$
$\Rightarrow \alpha = 4, \beta = 9$ and $\gamma = -3$
$\therefore$ Coordinate of other end of diameter are (4,9,-3)

Question330

The image of the point (-1,3,4) in the plane $x - 2y = 0$ is

[2006]

Options:

A. $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$

B. (15,11,4)

C. $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$

D. None of these

Answer: D

Solution:

Solution:
Let (α, β, γ) be the image, then mid point of (α, β, γ) and (-1,3,4) must lie on $x-2y=0$
$\therefore \frac{\alpha-1}{2}-2\left(\frac{\beta+3}{2}\right)=0$
$\therefore \alpha-1-2\beta-6=0 \Rightarrow \alpha-2\beta=7$ .....(1)
Also line joining (α, β, γ) and (-1,3,4) should be parallel to the normal of the plane $x-2y=0$
$\therefore \frac{\alpha+1}{1}=\frac{\beta-3}{-2}=\frac{\gamma-4}{0}=\lambda$
$\Rightarrow \alpha=\lambda-1, \beta=-2\lambda+3, \gamma=4$ .....(2)
From (1) and (2)
$\alpha=\frac{9}{5}, \beta=-\frac{13}{5}, \gamma=4$
None of the option matches.

Question331

If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is [2005]

Options:

A. (-1,2)

B. (-1,-2)

C. (1,-2)

D. $\left(1, -\frac{1}{2}\right)$

Answer: C

Solution:

Solution:

a, b, c are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

$\therefore \frac{x}{a} + \frac{y}{a} + \frac{1}{c} = 0$ passes through (1,-2)

Question332

The angle between the lines 2x = 3y = −z and 6x = −y = −4z is [2005]

Options:

A. 0°

B. 90°

C. 45°

D. 30°

Answer: B

Solution:

Solution:

The given lines are $2x = 3y = -z$

or $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ [Dividing by 6]

and $6x = -y = -4z$

or $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ [Dividing by 12]

∴ Angle between two lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{3.2 + 2\,.\,(-12) + (-6)\,.\,(-3)}{\sqrt{3^2 + 2^2 + (-6)^2}\,\sqrt{2^2 + (-12)^2 + (-3)^2}}$$
$$= 6 - 24 + 18\sqrt{49}\sqrt{157} \;= 0 \Rightarrow \theta = 90°$$

Question333

The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$ and the plane $\vec{r}\,.\,\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$ is
[2005]

Options:

A. $\frac{10}{9}$

B. $\frac{10}{3\sqrt{3}}$

C. $\frac{3}{10}$

D. $\frac{10}{3}$

Answer: B

Solution:

Solution:
The given line is $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$
and the plane is $\vec{r}\,.\,\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$
$\Rightarrow x + 5y + z = 5$
Required distance $= \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$
$= \left|\frac{2 - 10 + 3 - 5}{\sqrt{1 + 25 + 1}}\right| = \frac{10}{3\sqrt{3}}$

Question334

If the angle theta between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that$\sin\theta=\frac{1}{3}$ then the value of λ is
[2005]

Options:

A. $\frac{5}{3}$

B. $\frac{-3}{5}$

C. $\frac{3}{4}$

D. $\frac{-4}{3}$

Answer: A

Solution:

Solution:
Let θ is the angle between line and plane then

$$\sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$=\frac{\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left(2\hat{i}-\hat{j}+\sqrt{\lambda}\hat{k}\right)}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow\sin\theta=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3}\Rightarrow 4\lambda=5+\lambda$$

$$\Rightarrow\lambda=\frac{5}{3}$$

Question335

The plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius
[2005]

Options:

A. 3

B. 1

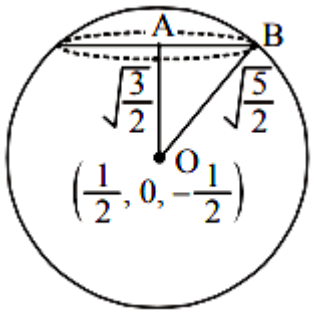
C. 2

D. $\sqrt{2}$

Answer: B

Solution:

Solution:



Centre of sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ and radius of sphere

$= \sqrt{\frac{1}{4}+\frac{1}{4}+2} = \sqrt{\frac{5}{2}}$

Perpendicular distance OA of centre from $x + 2y - z = 4$ is given by

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

$\therefore$ radius of circle $AB = \sqrt{OB^2 - OA^2} = \sqrt{\frac{5}{2}-\frac{3}{2}} = 1$

Question336

If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals [2005]

Options:

A. -1

B. 1

C. -2

D. 2

Answer: C

Solution:

Solution:
Plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the line joining the centres of spheres
$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and
$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$
respectively centre of spheres are $c_1(-3, 4, 1)$ and $c_2(5, -2, 1)$. Mid point of c_1c_2 is (1,1,1)
Satisfying this in the equation of plane, we get
$2a - 3a + 4a + 6 = 0$
$\Rightarrow a = -2.$

Question337

A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y -axis, is such that$\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
[2004]

Options:

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{3}{5}$

D. $\frac{2}{3}$

Answer: C

Solution:

Solution:
As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$
$\therefore \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$
$\because 2\cos^2\theta = 1 - \cos^2\theta$
$\Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)
$\Rightarrow 2\cos^2\theta = 3 - 3\cos^2\theta$
$\therefore \cos^2\theta = \frac{3}{5}$

Question338

If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are co-planar, then λ equals.
[2004]

Options:

A. 0

B. -1

C. $-\frac{1}{2}$

D. -2

Answer: D

Solution:

Solution:
The given lines are

$$x - 1 = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s \quad \text{.....(1)}$$

and $$2x = y - 1 = \frac{z-2}{-1} = t \quad \text{.....(2)}$$

The lines are coplanar, if

$$\begin{vmatrix} 0-1 & 1-(-3) & 2-1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

Apply $c_2 \to c_2 + c_3$; $\begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow -5\left(-1-\frac{\lambda}{2}\right)=0 \Rightarrow \lambda = -2$

Question339

A line with direction cosines proportional to 2,1,2 meets each of the lines x = y + a = z and x + a = 2y = 2z. The co-ordinates of each of the points of intersection are given by [2004]

Options:

A. (2a, 3a, 3a), (2a, a, a)

B. (3a, 2a, 3a), (a, a, a)

C. (3a, 2a, 3a), (a, a, 2a)

D. (3a, 3a, 3a), (a, a, a)

Answer: B

Solution:

Solution:
Let a point on the line $x = y + a = z = \lambda$ is
$(\lambda, \lambda - a, \lambda)$ and a point on the line
$x + a = 2y = 2z = \mu$ is $\left(\mu - a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then Direction ratio of the line joining these points are
$\lambda - \mu + a, \lambda - a - \frac{\mu}{2}, \lambda - \frac{\mu}{2}$
If it respresents the required line whose $d.r$ be $2, 1, 2$, then
$\frac{\lambda - \mu + a}{2} = \frac{\lambda - a - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$
on solving we get $\lambda = 3a, \mu = 2a$
$\therefore$ The required points of intersection are
$(3a, 3a - a, 3a)$ and $\left(2a - a, \frac{2a}{2}, \frac{2a}{2}\right)$
or $(3a, 2a, 3a)$ and (a, a, a)

Question340

Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
[2004]

Options:

A. $\frac{9}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{3}{2}$

Answer: C

Solution:

Solution:
The planes are $2x + y + 2z - 8 = 0$ .....(1)
and $4x + 2y + 4z + 5 = 0$
or $2x + y + 2z + \frac{5}{2} = 0$ .....(2)
Since, both planes are parallel
∴ Distance between (1) and (2)

$$= \left| \frac{\frac{5}{2} + 8}{\sqrt{2^2 + 1^2 + 2^2}} \right| = \left| \frac{21}{2\sqrt{9}} \right| = \frac{7}{2}$$

Question341

The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane
[2004]

Options:

A. $2x - y - z = 1$

B. $x - 2y - z = 1$

C. $x - y - 2z = 1$

D. $x - y - z = 1$

Answer: A

Solution:

Solution:
Given that, the equations of spheres are
$S_1 : x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
$S_2 : x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
We know that eqn. of intersection plane be
$S_1 - S_2 = 0 \Rightarrow 10x - 5y - 5z - 5 = 0$
$\Rightarrow 2x - y - z = 1$

Question342

The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if [2003]

Options:

A. k = 3 or -2

B. k = 0 or -1

C. k = 1 or -1

D. k = 0 or -3 .

Answer: D

Solution:

Solution:

Two planes are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$\Rightarrow 1[2 + 2k - (k + 2)(1 - k)] = 0$
$\Rightarrow 2 + 2k - (-k^2 - k + 2) = 0$
$k^2 + 3k = 0 \Rightarrow k(k + 3) = 0$
or $k = 0$ or -3

Question343

The two lines x = ay + b , z = cy + d and x = a'y + b', z = c'y + d' will be perpendicular, if and only if
[2003]

Options:

A. aa' + cc' + 1 = 0

B. aa' + bb' + cc' + 1 = 0

C. aa' + bb' +cc' = 0

D. (a + a') (b + b') +(c + c') = 0.

Answer: A

Solution:

Solution:
$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}; \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}.$
For perpendicularity of lines,
$aa' + 1 + cc' = 0$

Question344

Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′ from the origin then [2003]

Options:

A. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

B. $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

C. $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

D. $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

Answer: A

Solution:

Solution:

Equation of planes in intercept form be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ & $\frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

Question345

The radius of the circle in which the sphere$x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane$x+2y+2z+7=0$ is [2003]

Options:

A. 4

B. 1

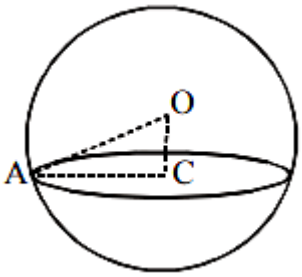
C. 2

D. 3

Answer: D

Solution:

Solution:



Centre of sphere $=(-1, 1, 2)$
Radius of sphere $\sqrt{1+1+4+19}=5$
Perpendicular distance from centre to the plane

$$OC = d = \left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = \frac{12}{3} = 4$$

In right, ΔAOC

$$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9$$

$$\Rightarrow AC = 3$$

Question346

The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is [2003]

Options:

A. 39

B. 26

C. $11\frac{4}{13}$

D. 13 .

Answer: D

Solution:

Solution:
Centre of sphere be (-2,1,3) and radius 13
We know that,
Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere - radius

$$= \left| \frac{-2 \times 12 + 4 \times 1 + 3 \times 3 - 327}{\sqrt{144 + 9 + 16}} \right| - 13$$

$$= 26 - 13 = 13$$

Question347

The d.r. of normal to the plane through (1,0,0),(0,1,0) which makes an angle $\pi/4$ with plane $x + y = 3$ are
[2002]

Options:

A. $1, \sqrt{2}, 1$

B. $1, 1, \sqrt{2}$

C. 1,1,2

D. $\sqrt{2}, 1, 1$

Answer: B

Solution:

Solution:
Equation of plane through (1,0,0) is

$$a(x-1) + by + cz = 0 \quad \text{.....(i)}$$

It is also passes through (0,1,0) .

$$\therefore -a + b = 0 \Rightarrow b = a$$

$$\cos 45^\circ = \frac{a + a}{\sqrt{2(2a^2 + c^2)}}$$

$$\Rightarrow 2a = \sqrt{2a^2 + c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a$$

So d.r of normal are a, a $\sqrt{2}$a i.e. $1, 1, \sqrt{2}$.

Question348

A plane which passes through the point (3,2,0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is

[2002]

Options:

A. $x-y+z=1$

B. $x+y+z=5$

C. $x+2y-z=1$

D. $2x-y+z=5$

Answer: A

Solution:

Solution:
Since the point (3,2,0) lies on the given line
$\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$
∴ There can be infinite many planes passing through this line. We observed that only option (a) is satisfied by the coordinates of both the points (3,2,0) and (4,7,4)
$\therefore x-y+z=1$ is the required plane.